2024-2025
Fall Semester

Course of Power System Analysis

The Synchronous Machine

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Outline

Description of the synchronous machine

Three phase generation

Synchronous reactance and equivalent circuits

Real and reactive power control

The two-axis machine model

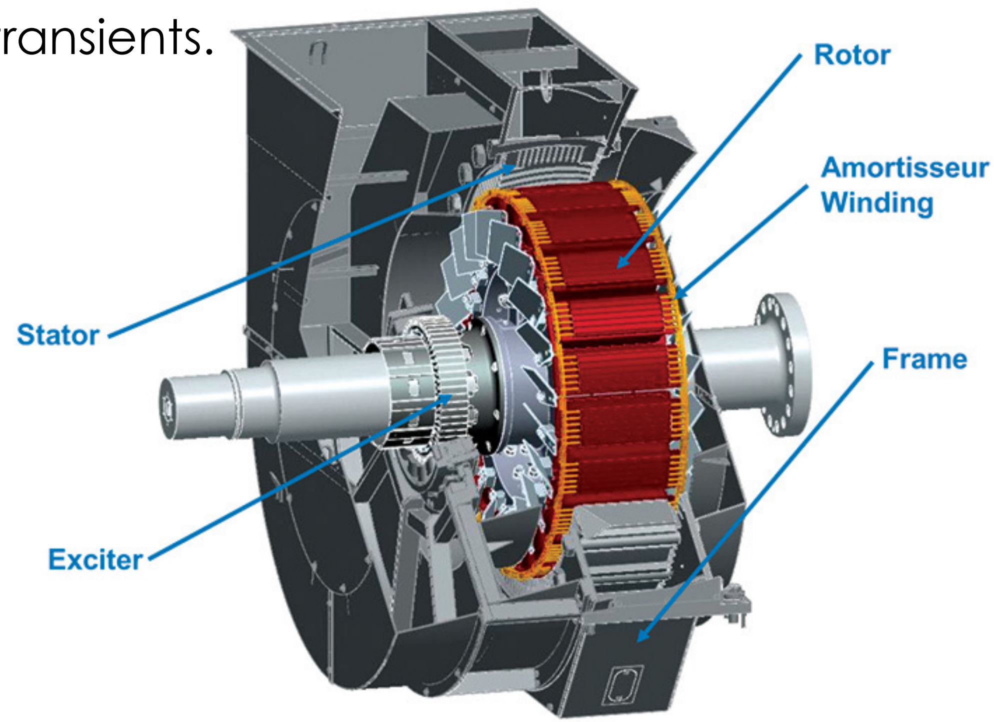
Transient and sub-transient effects

Description of the synchronous machine

The **synchronous machine** as an AC power generator, driven by a turbine, **converting mechanical into electrical energy.** This machine **is the major electric power generating source throughout the world.** When operating as a motor, the machine converts electrical energy to mechanical energy.

Our interests refer to the applications of the synchronous machine within a large interconnected power system, mainly considering the following conditions:

- steady-state;
- electromechanical transients.



Description of the synchronous machine

The two principal parts or a synchronous machine are ferromagnetic structures:

- **The stationary part:**
  - Called ***stator*** or ***armature***
  - Essentially a hollow cylinder
  - Has longitudinal slots with coils of the *armature windings*.
  - These windings carry the current supplied to an electrical load by a generator (or received from an ac supply by a motor)

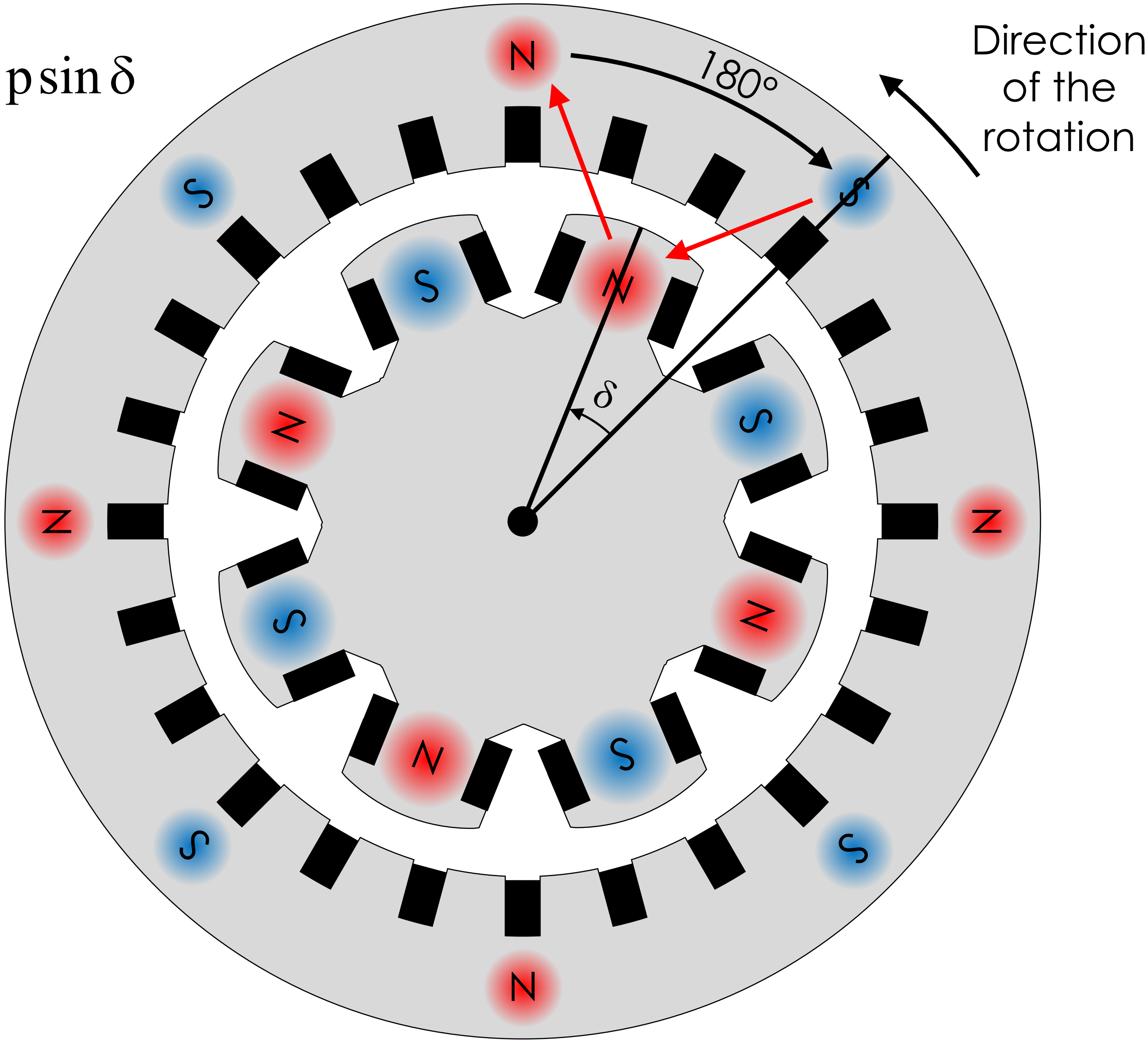
- **The rotating part:**
  - Called ***rotor***
  - Mounted on the shaft and **rotates** inside the hollow stator.
  - The winding on the rotor is called the ***field winding***
  - The field winding is supplied with DC current.

The resultant flux across the air gap between the stator and rotor generates voltages in the coils of the armature windings and provides the electromagnetic torque between the stator and rotor.

ELECTROMAGNETIC INDUCTION

Synchronous machine operating as a generator

$$T_{em} = k\,\hat{B}_s\hat{B}_r\,p\sin\delta$$



Description of the synchronous machine

The DC current is supplied to the field winding by an exciter, which may be:

- A generator mounted on the same shaft
- A separate DC source connected to the field winding through brushes bearing on slip rings (as in Fig. 1).

Large AC generators usually have exciters consisting of an AC source with solid-state rectifiers.

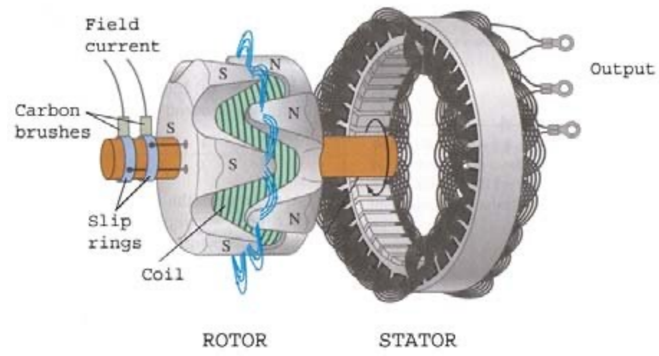


Fig. 1
Exploded view of
a synchronous machine

If the machine is a **generator**, the shaft is driven by a **prime mover**, which is usually a steam, gas or hydraulic turbine. As we have seen before, the electromagnetic torque developed in the generator when it delivers power opposes the torque of the prime mover.

When the machine operates as a motor, the **electromagnetic torque** developed in the machine (except for core and friction losses) is converted to the shaft torque which **drives the mechanical load**.

Description of the synchronous machine

Figure 2 shows the simplified cross-section view of a two-pole cylindrical machine of a generator called **round-rotor machine (or nonsalient rotor).**

- The field winding (rotor), indicated by the *f*-coil, gives rise to two **poles N and S** as marked.
- The axis of the field poles is called the **direct axis** or simply the ***d*-axis**
- The centerline of the interpolar space is called the **quadrature axis** or simply the ***q*-axis**.
- The positive direction along the ***d***-axis leads the positive direction along ***q***-axis by 90° (as shown in the figure).

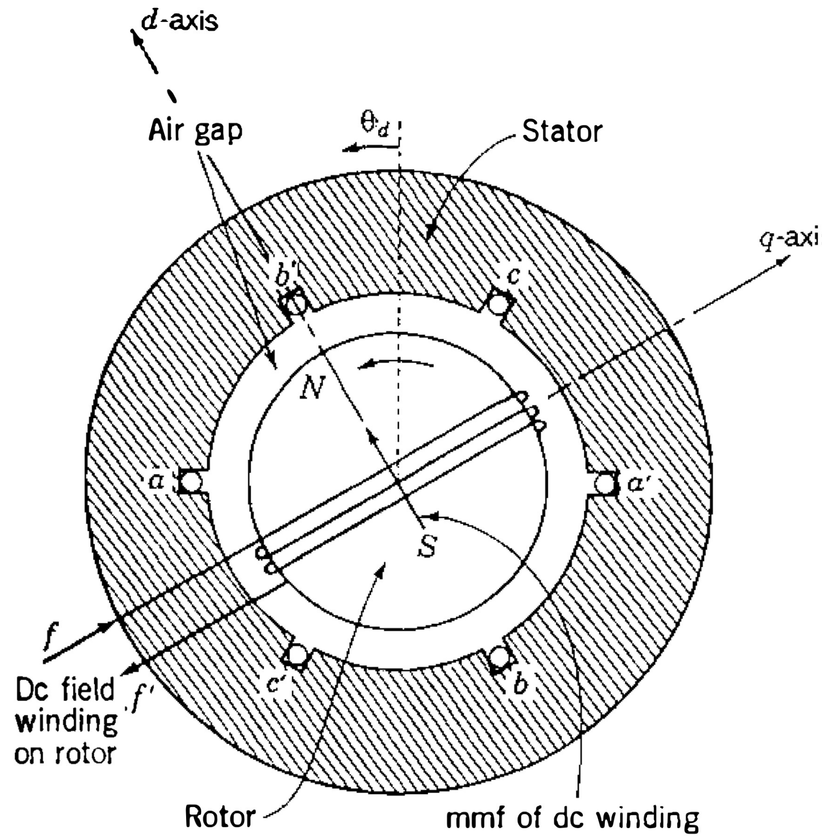
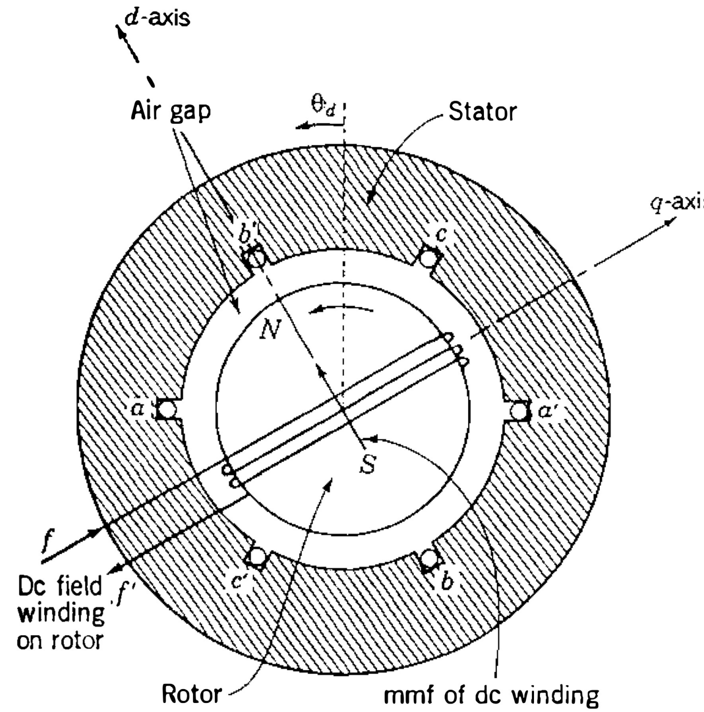


Figure 2
Elementary three-phase AC generator

Description of the synchronous machine



Stator

- The opposite sides of a coil, are in slots ***a*** and ***a'***, 180° apart.
- For a three phase machine, identical coils are in slots ***b*** and ***b'***, and slots ***c*** and ***c'***.
- Coil sides in slots ***a***, ***b***, ***c*** are 120° apart.

The conductors shown in the slots indicate a coil of only one turn, but such a coil **may have many turns** and is usually in series with identical coils in adjacent slots to form a winding having ends designated ***a*** and ***a***'.

Rotor

In the actual **machine the winding has a large number of turns** distributed in slots around the circumference of the rotor.
The strong magnetic field produce links the stator coils to induce voltage in the armature windings as the shaft is turned by the prime mover.

Description of the synchronous machine

Figure 3 shows a **salient-pole** machine which has four poles (i.e., 2 pole pairs).

The **different numbers of poles**, with respect to Figure 2, has the following consequences:

- The opposite sides of an armature coil are 90° apart. So, there are two coils for each phase.
- Coil sides ***a*, *b*,** and ***c*** of adjacent coils are 60° apart.
- The two coils of each phase may be connected in series or in parallel.

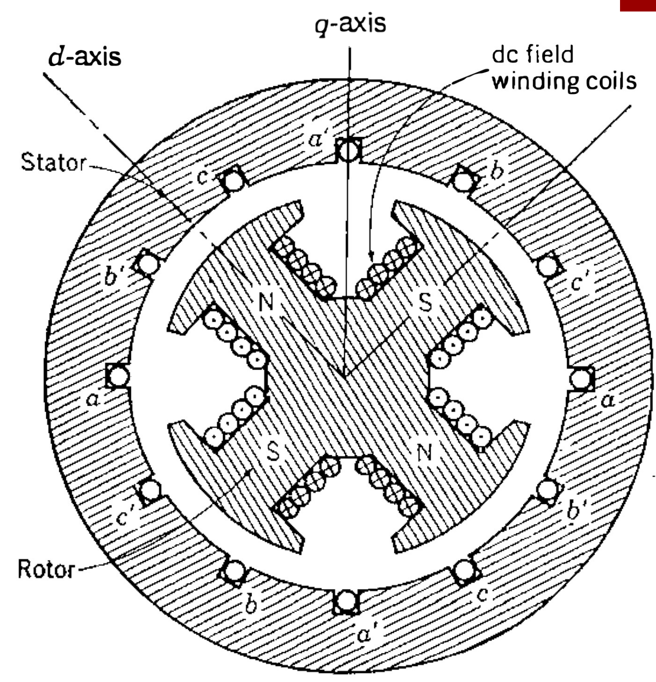


Figure 3
Cross section of an elementary stator and salient-pole rotor

Although not shown in Fig. 2, salient-pole machines usually have damper windings, which consist of short-circuited copper bars through the pole. The purpose of the damper winding is to reduce the mechanical oscillations of the rotor about synchronous speed. This aspect will be discussed later.

Description of the synchronous machine

Obs #1: the windings of the polyphase synchronous machine constitute a group of **inductively coupled electric circuits**, **some of which rotate relative to others** so that mutual inductances are variable.

Hp#1: Only linear magnetic circuits are considered. Therefore, the saturation of magnetic parts is neglected.

Obs#2: Hp#1 allows us to refer separately to the flux and flux linkages produced by a **magnetomotive force (mmf)**. Pay attention that in any electric machine there exists only the net physical flux due to the resultant mmf of all the magnetizing sources (i.e., the rotor and the stator one).

Description of the synchronous machine

In the **two-pole** machine **one cycle** of voltage is generated for each **revolution** of the two-pole rotor while in the **four-pole** machine **two cycles** are generated in each coil **per revolution**, as shown if Figure 4.

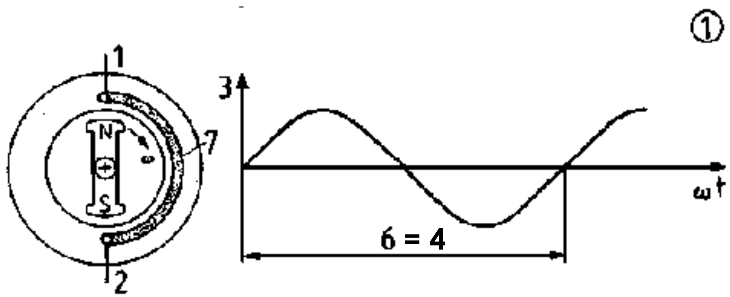


Figure 4.1 Two-pole generator

(1) Winding beginning
(2) Winding end
(3) Voltage
(4) A cycle of the voltage
(6) A rotation of the shaft = 1 cycle
(7) Coil

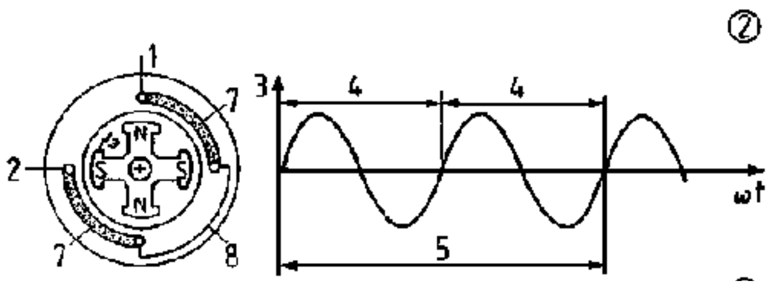


Figure 4.2 Four-pole generator

(1) Winding beginning
(2) Winding end
(3) Voltage
(4) A cycle of the voltage
(5) A rotation of the shaft = 2 cycles
(7) Coil
(8) Coil connection

Description of the synchronous machine

- In the two-pole machine one cycle of voltage is generated for each revolution of the two-pole rotor.
- In the four-pole machine two cycles are generated in each coil per revolution.

Since the number of cycles per revolution equals the number of pairs of poles, the frequency of the generated voltage is

$$f = \frac{P}{2}\frac{N}{60} = p\frac{N}{60} = \frac{P}{2} f_m \qquad (1.1)$$

Where:

f = electrical frequency in Hz

P = number of poles (p = number of pairs of poles)

N = rotor speed in rpm

$f_m = N/60$ = mechanical frequency in revolution per seconds (Hz)

Eq.(1.1) tells us that there is a strict equality constraint between the mechanical frequency (i.e. rotation speed) of the machine and its electrical frequency. A two-pole, 50 Hz machine operates at 3000 rpm, whereas a four-pole machine operates at 1500 rpm.

Usually, fossil-fired steam turbo-generators are two-pole machines, whereas hydro-generating units are slower machines with many pole pairs.

Description of the synchronous machine

Since one cycle of voltage (360° of the voltage wave) is generated every time a pair of poles passes a coil, we must distinguish between **electrical degrees used to express voltage** and current and **mechanical degrees used to express the position of the rotor**.

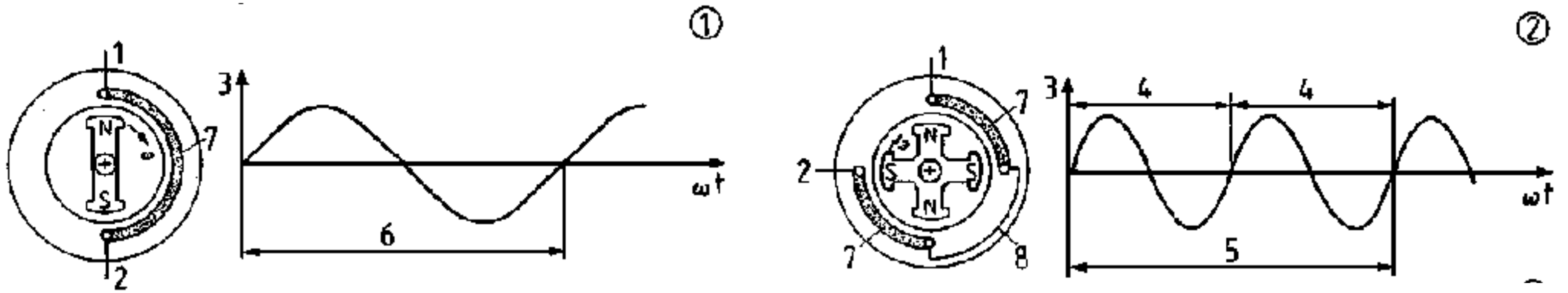


Figure 4

- In a **two-pole** machine electrical and mechanical degrees are equal.
- In a **four-pole** machine, therefore, two cycles, or 720° electrical degrees, are produced per revolution of 360° mechanical degrees.

In any machine the number of electrical degrees or radians equals $P/2 = p$ times the number of mechanical degrees or radians, as can be seen from Eq. (1.1) by multiplying both sides by 2π.

All angular measurements are going to be expressed in electrical degrees and the direct axis always leads the quadrature axis by 90 electrical degrees in the counter-clockwise direction of rotation

Outline

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Three phase generation

Synchronous reactance and equivalent circuits

Real and reactive power control

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Transient and sub-transient effects

Three-phase generation

Armature winding

- Coils $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ represent the three **armature windings** on the stator of the round-rotor machine
- Concentrated coil $\boldsymbol{f}$ represents the distributed **field winding** on the rotor
- The three stationary armature coils **are identical** in every respect and each has one of its two terminals connected to a common point 'O'.
- The axis of coil $\boldsymbol{a}$ is chosen at $\theta_d = 0°$
- Counter-clock wise around the air gap the axes of the **b-coil** is chosen at $\boldsymbol{\theta_d = 120°}$
- Counter-clock wise around the air gap the axes of the **c-coil** is chosen at $\boldsymbol{\theta_d = 240°}$

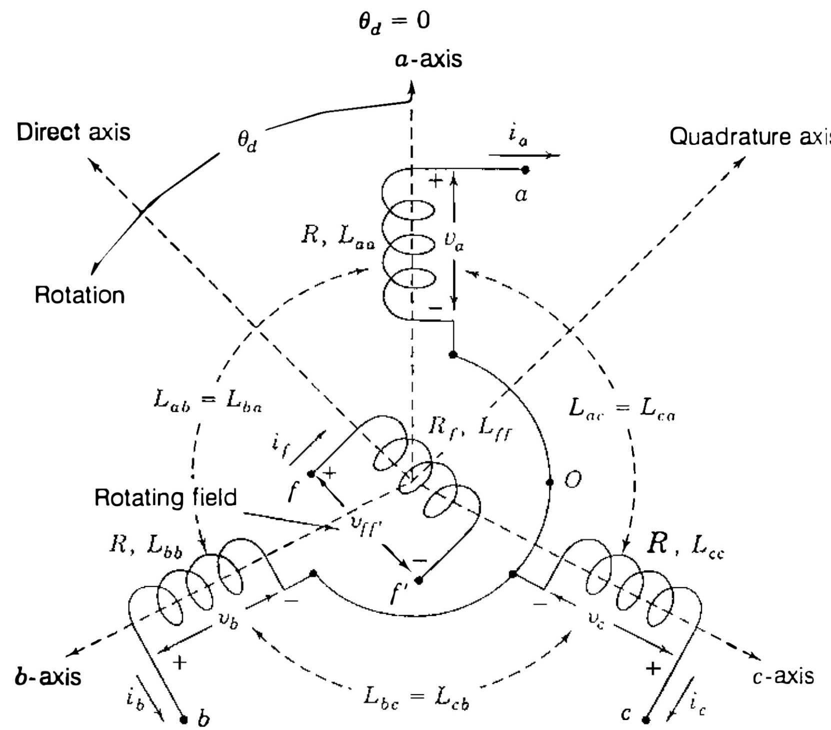


Figure 5

Idealized three-phase generator showing identical armature coils $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ and field coil $\boldsymbol{f}$. Direct axis leads quadrature axis by 90° in the anticlockwise direction of rotation.

Three-phase generation

Hypothesis:

1. Coils $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ have **self-inductance** $\boldsymbol{L_s}$ respectively equal to the self-inductances L_{aa}, L_{bb}, L_{cc} of the distributed armature windings which the coils represent so that:

$$L_s = L_{aa} = L_{bb} = L_{cc}$$

2. The **mutual inductances** $\boldsymbol{L_{ab}}$, $\boldsymbol{L_{bc}}$ and $\boldsymbol{L_{ac}}$ between each adjacent pair of concentrated coils are negative constants denoted by $-M_S$:

$$-M_s = L_{ab} = L_{bc} = L_{ca}$$

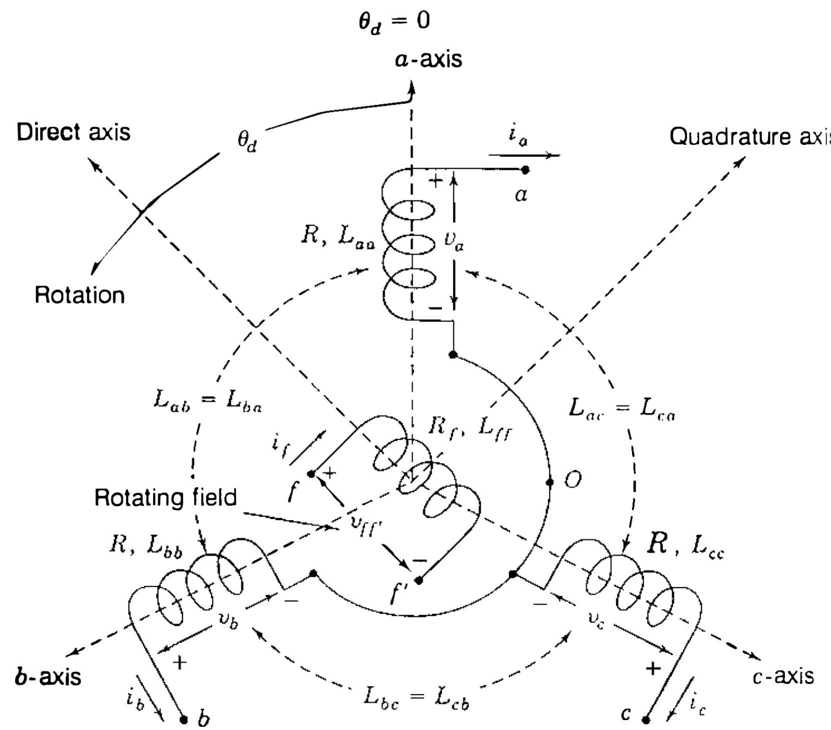


Figure 5

3. The **field coil** has a constant **self-inductance** $\boldsymbol{L_{ff}}$

4. Currents i_a, i_b, i_c are a balanced three-phase set of currents: $i_a + i_b + i_c = 0$

5. The **mutual inductance between the field coil** $\boldsymbol{f}$ **and each of the stator coils varies with the rotor position** θ_d as a cosinusoidal function with maximum value M_f so that:

$$L_{af} = M_f \cos\theta_d \qquad L_{bf} = M_f \cos(\theta_d - 120°) \qquad L_{cf} = M_f \cos(\theta_d - 240°)$$

Three-phase generation

Flux linkages with each of the coils ***a, b, c*** and ***f*** are due to its own current and the currents in the three other coils.
Flux-linkage equations are therefore written for all four coils as follows:

Armature:

$$\begin{aligned}\lambda_a &= L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f \\ \lambda_b &= L_{ba}i_a + L_{bb}i_b + L_{bc}i_c + L_{bf}i_f \\ \lambda_c &= L_{ca}i_a + L_{cb}i_b + L_{cc}i_c + L_{cf}i_f\end{aligned} \tag{2.1}$$

Field:

$$\lambda_f = L_{af}i_a + L_{bf}i_b + L_{cf}i_c + L_{ff}i_f \tag{2.2}$$

Taking into consideration the previous hypothesis, the flux linkage equations (2.1),(2.2) can be transformed in such a way:

Armature:

$$\begin{aligned}\lambda_a &= L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f = L_s i_a - M_s(i_b + i_c) + L_{af}i_f \\ \lambda_b &= L_{ba}i_a + L_{bb}i_b + L_{bc}i_c + L_{bf}i_f = L_s i_b - M_s(i_a + i_c) + L_{bf}i_f \\ \lambda_c &= L_{ca}i_a + L_{cb}i_b + L_{cc}i_c + L_{cf}i_f = L_s i_c - M_s(i_a + i_b) + L_{cf}i_f\end{aligned} \tag{2.3}$$

Three-phase generation

Armature winding

Remember that, if we consider i_a, i_b, i_c as a **balanced three-phase set of currents**, we have:

$$i_a + i_b + i_c = 0$$

$$i_a = -(i_b + i_c)$$
$$i_b = -(i_a + i_c)$$
$$i_c = -(i_a + i_b)$$

By introducing these last equations into Eq. (2.3) we obtain:

$$\begin{aligned} \lambda_a &= (L_s + M_s) i_a + L_{af} i_f \\ \lambda_b &= (L_s + M_s) i_b + L_{bf} i_f \\ \lambda_c &= (L_s + M_s) i_c + L_{cf} i_f \end{aligned} \qquad (2.4)$$

STEADY STATE HYPOTESYS:

- The current i_f is DC with a constant value I_f
- The field rotates at constant angular velocity ω so that for the two-pole machine

$$\frac{d\theta_d}{dt} = \omega \qquad \text{and} \qquad \theta_d = \omega t + \theta_{d0}$$

Where θ_{d0} can be arbitrarily chosen at $t = 0$.

Three-phase generation

Armature winding

Eqs. (2.4), by knowing that:

$L_{af} = M_f \, cos\theta_d$ $\qquad$ $L_{bf} = M_f \, cos(\theta_d - 120°)$ $\qquad$ $L_{cf} = M_f \, cos(\theta_d - 240°)$

and **considering steady state condition** become:

$$\begin{aligned} \lambda_a &= (L_s + M_s)i_a + M_f I_f \cos(\omega t + \theta_{d0}) \\ \lambda_b &= (L_s + M_s)i_b + M_f I_f \cos(\omega t + \theta_{d0} - 120°) \\ \lambda_c &= (L_s + M_s)i_c + M_f I_f \cos(\omega t + \theta_{d0} - 240°) \end{aligned} \tag{2.5}$$

The first of these equations shows that $\boldsymbol{\lambda_a}$ has two flux-linkage components:

- One due to the **field current $\boldsymbol{I_f}$**
- One due to the ***armature current $\boldsymbol{i_a}$*** (which is flowing out of the machine stator to generate power towards the external load)

The same can be applied for the other two phases.

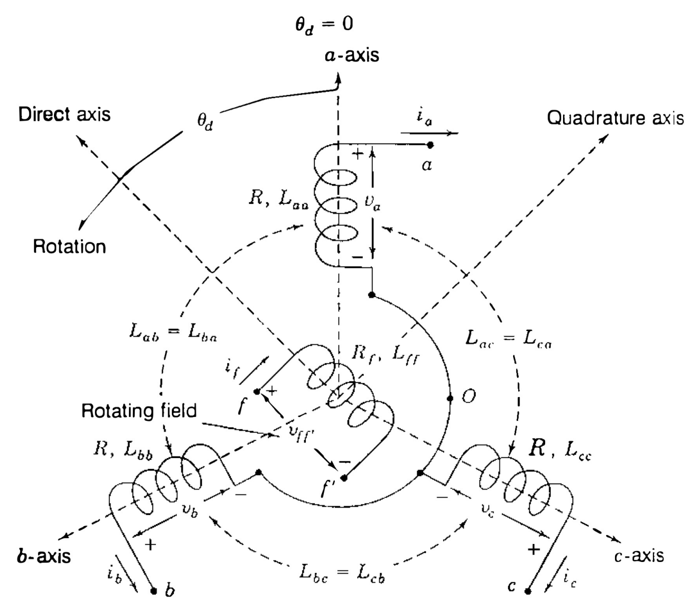


Figure 5

Three-phase generation

Armature winding

If coil $\boldsymbol{a}$ has resistance R, then the voltage drop v_a across the coil from terminal $\boldsymbol{a}$ to terminal O in Fig. 5 is given by:

$$v_a = -Ri_a - \frac{d\lambda_a}{dt} = -Ri_a - (L_s + M_s)\frac{di_a}{dt} + \omega M_f I_f \sin(\omega t + \theta_{d0}) \tag{2.6}$$

The negative signs apply because the machine is being treated as a generator. The last term of Eq. (2.6) represents an **internal electromotive force (emf)**, which we now call $e_{a'}$. This emf can be written as:

$$e_{a'} = \sqrt{2}|E_i|\sin(\omega t + \theta_{d0}) \tag{2.7}$$

where: $|E_i| = \frac{\omega M_f I_f}{\sqrt{2}}$

The action of the field current causes e_a' to appear across the terminals of the a-phase when i_a is zero, and so it is called by various names such as:

- No-load voltage
- Open-circuit voltage
- Synchronous internal voltage
- Generated emf of phase a

Three-phase generation

Armature winding

In Eqs.(2.6) and (2.7) **the angle $\boldsymbol{\theta_{d0}}$ indicates the position of the field winding** (and the $\boldsymbol{d}$-axis) relative to the $\boldsymbol{a}$-phase **at $\boldsymbol{t=0}$**. Hence, $\boldsymbol{\delta} = \theta_{d0} - 90°$ **indicates the position of the $\boldsymbol{q}$-axis**, which is 90° behind the $\boldsymbol{d}$-axis in Fig. 5.

For later convenience we now set $\theta_{d0} = \delta + 90°$ and then we have:

$$\theta_d = \omega t + \theta_{\mathrm{d}0} = \omega t + \delta + 90° \qquad (2.8)$$

where θ_d, ω and δ have consistent units of angular measurement. Substituting from Eq. (2.8) into Eq. (2.7) and noting that $sin(\alpha + 90°) = cos\,\alpha$ we obtain for the open-circuit voltage of phase a:

$$e_{a'} = \sqrt{2}|E_i|\cos(\omega t + \delta) \qquad (2.9)$$

The terminal voltage v_a of Eq. (2.6) is then given by:

$$\boxed{v_a = -Ri_a - (L_s + M_s)\frac{di_a}{dt} + \sqrt{2}|E_i|\cos(\omega t + \delta)} \qquad (2.10)$$

This equation corresponds to the $\boldsymbol{a}$-phase circuit of Fig. 6 in which the no-load voltage $\boldsymbol{e_{a'}}$ is the source and the external load is balanced across all three phases.

Three-phase generation

Armature winding

It is possible to apply the same for the no-load voltages $e_{b'}$, and $e_{c'}$ which lag $e_{a'}$ by 120° and 240°, respectively.
Hence, $\boldsymbol{e_{a'}, e_{b'}}$ **and** $\boldsymbol{e_{c'}}$ constitute a **balanced three-phase** set of emfs which give rise to balanced three-phase line currents, say:

$$\begin{aligned} i_a &= \sqrt{2}|I_a|\cos(\omega t + \delta - \theta_a) \\ i_b &= \sqrt{2}|I_a|\cos(\omega t + \delta - \theta_a - 120°) \\ i_c &= \sqrt{2}|I_a|\cos(\omega t + \delta - \theta_a - 240°) \end{aligned} \tag{2.11}$$

Where:

- $|I_a|$ is the rms value
- $\boldsymbol{\theta_a}$ **is the phase angle of the current** $\boldsymbol{i_a}$ **with respect to** $\boldsymbol{e_{a'}}$**.**

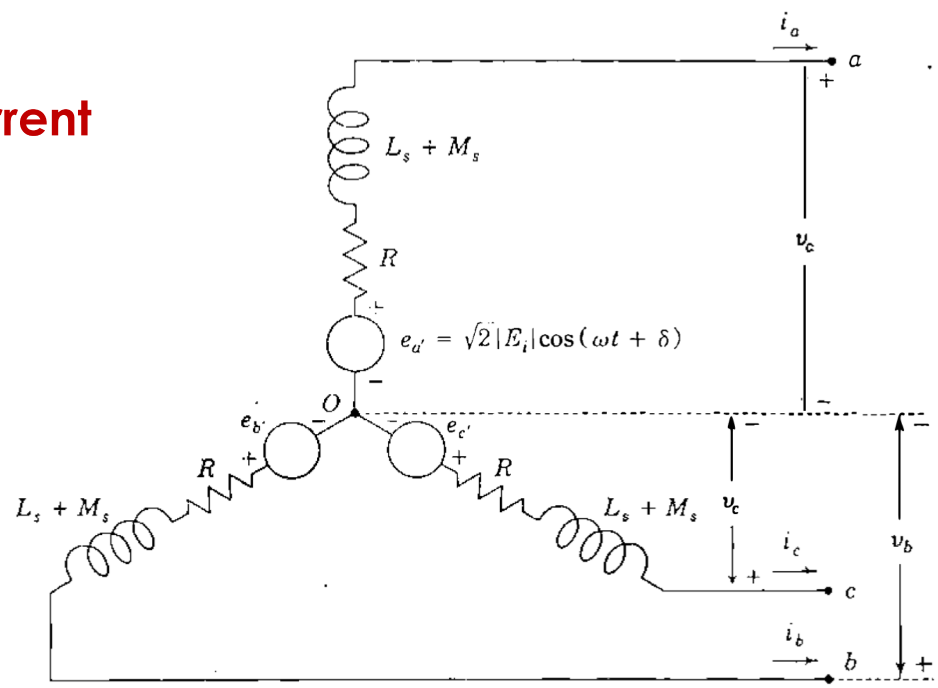


Figure 6
Armature equivalent circuit of the idealized three-phase generator showing balanced no-load voltages $e_{a'}, e_{b'}, e_{c'}$ in the steady state

Three-phase generation

Field winding

As previously done with the armature coil, the target is to find expressions for **flux linkage and current related to the field winding**. To do that, it is possible to start by substituting the definition of L_{af}, L_{bf}, L_{cf}:

$L_{af} = M_f \cos\theta_d$ $\quad$ $L_{bf} = M_f \cos(\theta_d - 120°)$ $\quad$ $L_{cf} = M_f \cos(\theta_d - 240°)$

into Eq. (2.2), which states that $\lambda_f = L_{af} i_a + L_{bf} i_b + L_{cf} i_c + L_{ff} i_f$, to yield:

$$\boxed{\lambda_f = L_{ff} I_f + M_f [i_a \cos\theta_d + i_b \cos(\theta_d - 120°) + i_c \cos(\theta_d - 240°)]} \qquad (2.12)$$

This expression for the flux will be simplified in the following slides.

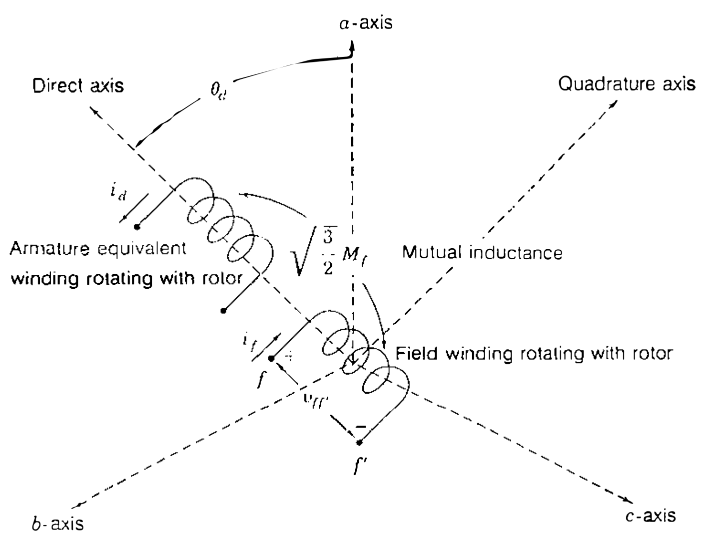


Figure 7

Representing the armature of the synchronous machine by a direct-axis winding of mutual inductance $\sqrt{3/2}M_f$ with the field winding. Both windings rotate together in synchronism.

Three-phase generation

Field winding

Knowing that: $\theta_d = \omega t + \delta + 90°$, the first term within the brackets Eq. of (2.12) can be written as:

$$\boldsymbol{i_a} \cos\theta_d = \boldsymbol{\sqrt{2}\,|I_a| \cos(\omega t + \delta - \theta_a)} \cos(\omega t + \delta + 90°) \tag{2.13}$$

Considering that: $2\cos\alpha\cos\beta \ = \cos(\alpha - \beta) + \cos(\alpha + \beta)$

$$\begin{aligned} i_a \cos\theta_d &= \frac{|I_a|}{\sqrt{2}}\{-\sin\theta_a - \sin[2(\omega t + \delta) - \theta_a]\} \\ i_b \cos(\theta_d - 120°) &= \frac{|I_a|}{\sqrt{2}}\{-\sin\theta_a - \sin[2(\omega t + \delta) - \theta_a - 120°]\} \\ i_c \cos(\theta_d - 240°) &= \frac{|I_a|}{\sqrt{2}}\{-\sin\theta_a - \sin[2(\omega t + \delta) - \theta_a - 240°]\} \end{aligned} \tag{2.14}$$

The terms involving $2\omega t$ in Eqs. (2.14) are balanced second harmonic sinusoidal quantities which sum to zero at each point in time. Hence, adding the bracketed terms of Eq. (2.14) together, we obtain:

$$[i_a \cos\theta_d + i_b \cos(\theta_d - 120°) + i_b \cos(\theta_d - 240°)] = -\frac{3|I_a|}{\sqrt{2}} \sin\theta_a \tag{2.15}$$

Three-phase generation

Field winding

If we substitute Eq. (2.15) into Eq.(2.12) for the flux linkages λ_f we obtain:

$$\boxed{\lambda_f = L_{ff} I_f - \frac{3M_f|I_a|}{\sqrt{2}} \sin\theta_a = L_{ff} I_f + \sqrt{3/2}\, M_f i_d} \tag{2.16}$$

where the DC current i_d is defined as:

$$i_d = -\sqrt{3}\, |I_a| \sin\theta_a \tag{2.17a}$$

or else:

$$i_d = \sqrt{2/3}\, [i_a \cos\theta_d + i_b \cos(\theta_d - 120°) + i_b \cos(\theta_d - 240°)] \tag{2.17b}$$

In general, the field winding with resistance R_f and entering current i_f has terminal voltage $v_{ff'}$ given by:

$$\boxed{v_{ff'} = R_f i_f + \frac{d\lambda_f}{dt}} \tag{2.18}$$

Because λ_f is not varying with time in the steady state, the field voltage becomes $v_{ff'} = R_f I_f$ and $i_f = I_f$ can be supplied by a DC source.

Three-phase generation
Field winding

We now recall the obtained equation (2.16) for the flux linkages λ_f:

$$\lambda_f = L_{ff} I_f + \sqrt{3/2}\, M_f i_d \tag{2.16}$$

Observations:

- The **flux linkages with the field winding** due to the combination of i_a, i_b and i_c **do not vary with time**.
- We can regard those **flux linkages** as coming from the steady **DC current i_d** in a **fictitious DC circuit coincident with the d-axis**
- **The fictitious DC circuit is stationary with respect to the field circuit.**
- The two circuits rotate together in synchronism end have a **mutual inductance $\sqrt{3/2}\, M_f$.**

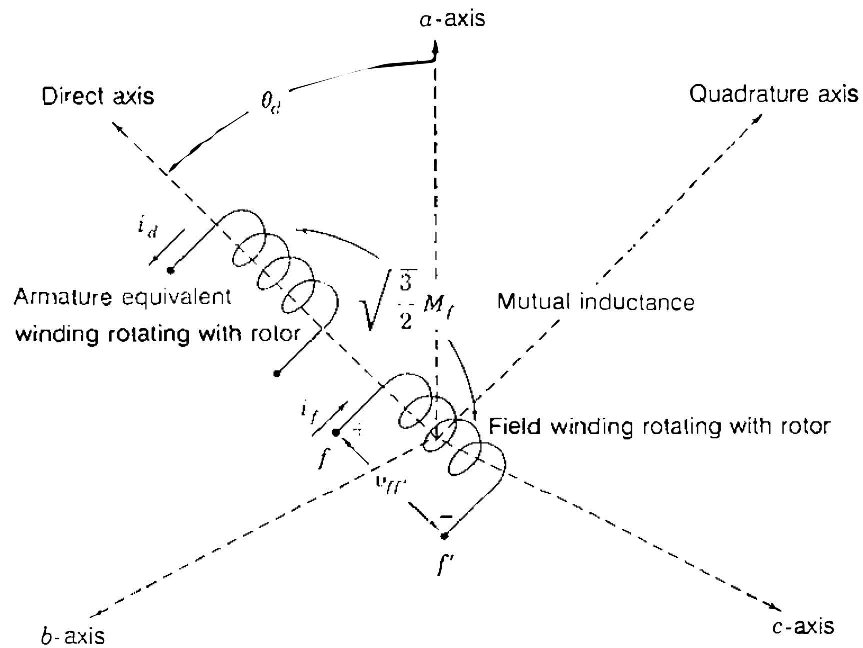


Figure 7

Three-phase generation
Summary

Hypothesis:
Balanced three-phase system + Steady state

Armature:

$$\lambda_a = (L_s + M_s)i_a + M_f I_f \cos(\omega t + \theta_{d0})$$
$$\lambda_b = (L_s + M_s)i_b + M_f I_f \cos(\omega t + \theta_{d0} - 120°) \qquad (2.5)$$
$$\lambda_c = (L_s + M_s)i_c + M_f I_f \cos(\omega t + \theta_{d0} - 240°)$$

$$v_a = -Ri_a - (L_s + M_s)\frac{di_a}{dt} + e_{a'} \qquad (2.10)$$
$$e_{a'} = \sqrt{2}|E_i|\sin(\omega t + \theta_{d0}) \qquad (2.7)$$
where $|E_i| = \frac{\omega M_f I_f}{\sqrt{2}}$

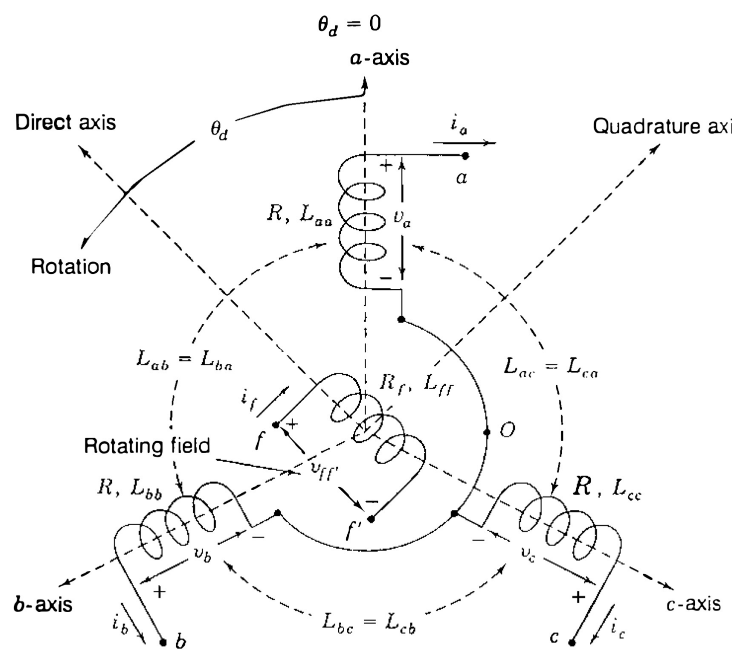
$$i_a = \sqrt{2}|I_a|\cos(\omega t + \delta - \theta_a)$$
$$i_b = \sqrt{2}|I_a|\cos(\omega t + \delta - \theta_a - 120°) \qquad (2.11)$$
$$i_c = \sqrt{2}|I_a|\cos(\omega t + \delta - \theta_a - 240°)$$

Field:

$$\lambda_f = L_{ff} I_f - \frac{3M_f|I_a|}{\sqrt{2}}\sin\theta_a = L_{ff} I_f + \sqrt{3/2}\, M_f i_d \qquad (2.16)$$

$$v_{ff'} = R_f i_f + \frac{d\lambda_f}{dt} \qquad (2.18)$$

$$i_d = -\sqrt{3}\,|I_a|\sin\theta_d \qquad (2.17)$$



Outline

Description of the synchronous machine

Three phase generation

Synchronous reactance and equivalent circuits

Real and reactive power control

The two-axis machine model

Transient and sub-transient effects

Synchronous reactance and equivalent circuits

The coupled-circuit model in Fig. 6 represents the idealized Y-connected round-rotor synchronous machine.

If the machine is rotating at synchronous speed ω and that the field current I_f is steady DC, the balanced three-phase circuit of Fig. 5 gives the steady-state operation of the machine.

The no-load voltages are the emfs $e_{a'}, e_{b'}$ and $e_{c'}$.

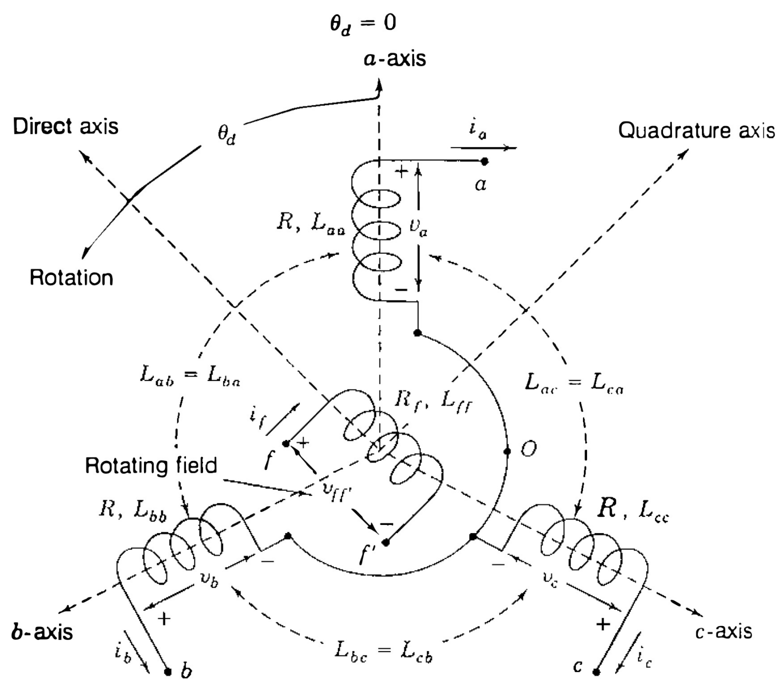


Figure 5

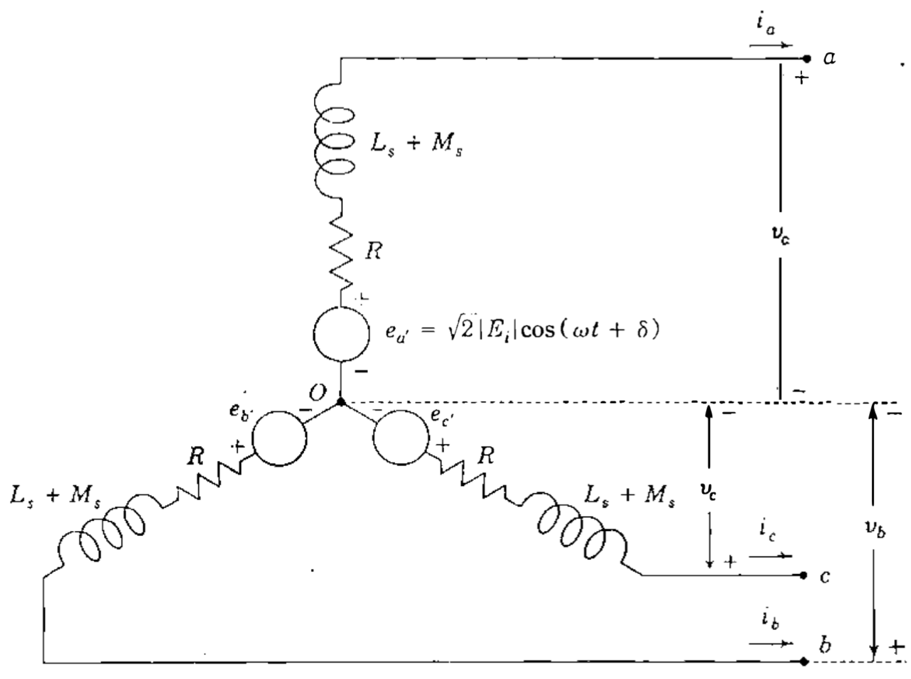


Figure 6

Synchronous reactance and equivalent circuits

The per-phase equivalent circuit with steady-state sinusoidal currents and voltages is shown in Fig. 8(a). Now if we recall Eq. (2.11):

$$i_a = \sqrt{2}|I_a| \cos(\omega t + \delta - \theta_a) \qquad (2.11)$$

we can note that the phase angle of the current i_a in Eq. (2.11) is chosen with respect to the no-load voltage e_a, of the $\boldsymbol{a}$-phase.

In practice, $\boldsymbol{e_a}$ **cannot be measured under load**, and so it is preferable to choose the terminal voltage v_a as reference and to measure the phase **angle of the current $\boldsymbol{i_a}$ with respect to $\boldsymbol{v_a}$.** Therefore, we define:

$$v_a = \sqrt{2}|V_a| \cos \omega t \qquad (3.1)$$

$$e_{a'} = \sqrt{2}|E_t| \cos(\omega t + \delta) \qquad (3.2)$$

$$i_a = \sqrt{2}|I_a| \cos(\omega t - \theta) \qquad (3.3)$$

Where $\theta = \theta_a - \delta$ is now the the angle of lag of i_a with respect to v_a.

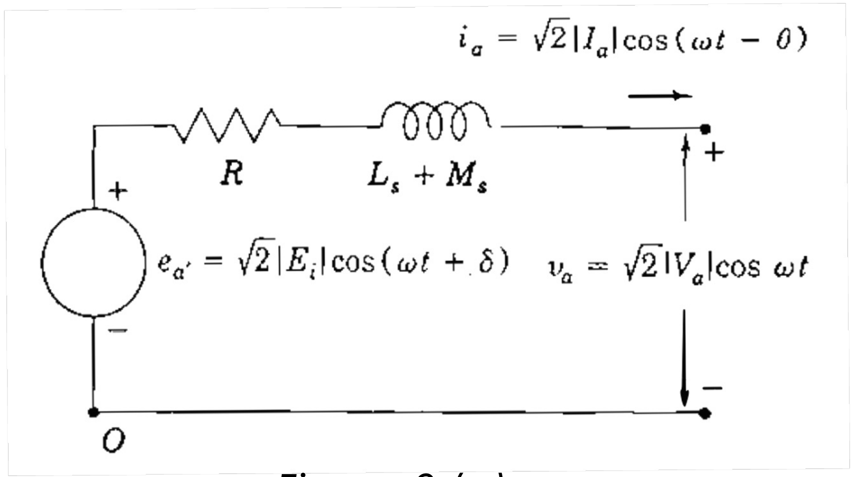


Figure 8 (a):

Equivalent circuit for reference phase $\boldsymbol{a}$ of the synchronous machine showing voltages and currents as cosinusoidal quantities.

Synchronous reactance and equivalent circuits

The phasor equivalents of Eqs. (3.1), (3.2) and (3.3), are:

$$\bar{V}_a = |V_a| \angle 0° \quad (3.4)$$

$$\bar{E}_{a'} = |E_t| \angle \delta° \quad (3.5)$$

$$\bar{I}_a = |I_a| \angle (-\theta)° \quad (3.6)$$

- When the current $\bar{I}_a$ leads $\bar{V}_a$ the angle θ is numerically negative
- When $\bar{I}_a$ lags $\bar{V}_a$, the angle θ is numerically positive

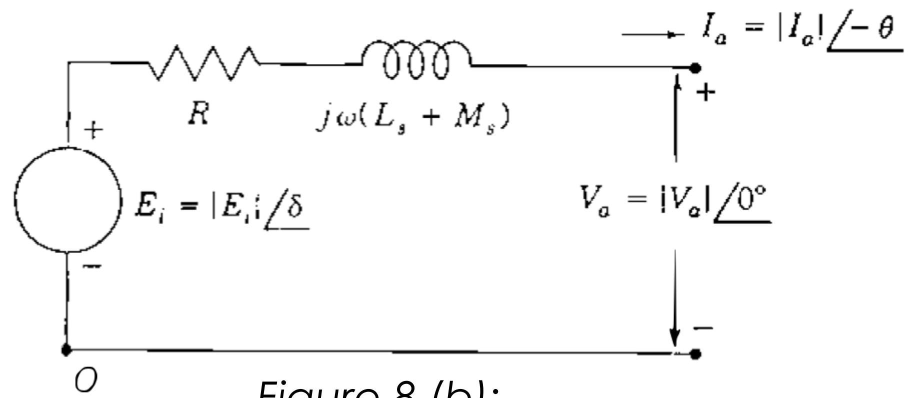


Figure 8 (b):

Equivalent circuit for reference phase $\boldsymbol{a}$ of the synchronous machine showing voltages and currents as phasor quantities.

The phasor-voltage equation is:

$$\boxed{\bar{V}_a = \underbrace{\bar{E}_i}_{\text{Generated at no load}} - \underbrace{R\bar{I}_a}_{\text{Due to armature resistance}} - \underbrace{j\omega L_s \bar{I}_a}_{\text{Due to armature self-reactance}} - \underbrace{j\omega M_s \bar{I}_a}_{\text{Due to armature mutual-reactance}} \quad (3.7)}$$

Since symmetrical conditions apply, phasor equations corresponding to Eq. (3.7) can be written for $\boldsymbol{b}$-phase and $\boldsymbol{c}$-phase as well with lags of $-120°$ and $240°$.

Synchronous reactance and equivalent circuits

$\bar{V}_a$	$=$	$\bar{E}_i$	$-$	$R\bar{I}_a$	$-$	$j\omega L_s\bar{I}_a$	$-$	$j\omega M_s\bar{I}_a$	(3.7)
		Generated at no load		Due to armature resistance		Due to armature self-reactance		Due to armature mutual-reactance	

The combined quantity $\omega(L_s + M_s)$ of Eq. (3.7) has the dimensions of reactance and is customarily called the **synchronous reactance X_d** of the machine. The **synchronous impedance $\bar{Z}_d$** of the machine is defined by

$$\bar{Z}_d = R + jX_d = R + j\omega(L_s + M_s) \tag{3.8}$$

and Eq. (3.7) then can be written in the more compact form

$$\bar{V}_a = \bar{E}_i - \bar{I}_a\bar{Z}_d = \bar{E}_i - \bar{I}_aR - j\bar{I}_aX_d \tag{3.9}$$

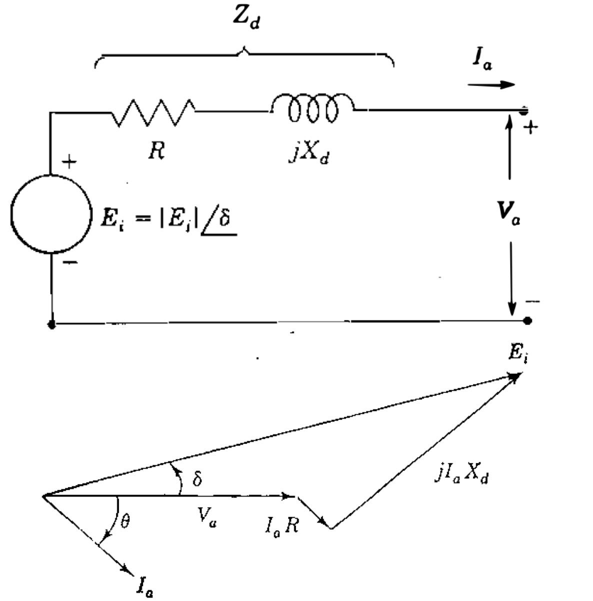
The equivalent circuit for the **synchronous motor** is identical to that of the generator, except that **the direction of $\bar{I}_a$ is reversed**, as shown in Fig. 9(b), which has the equation:

$$\bar{V}_a = \bar{E}_i + \bar{I}_a\bar{Z}_d = \bar{E}_i + \bar{I}_aR + j\bar{I}_aX_d \tag{3.10}$$

Synchronous reactance and equivalent circuits

Phasor diagrams for the previous equations are shown in Fig. 9. For the generator note that $\bar{E}_i$ always leads $\bar{V}_a$, and for the motor $\bar{E}_i$ always lags $\bar{V}_a$.

Generator

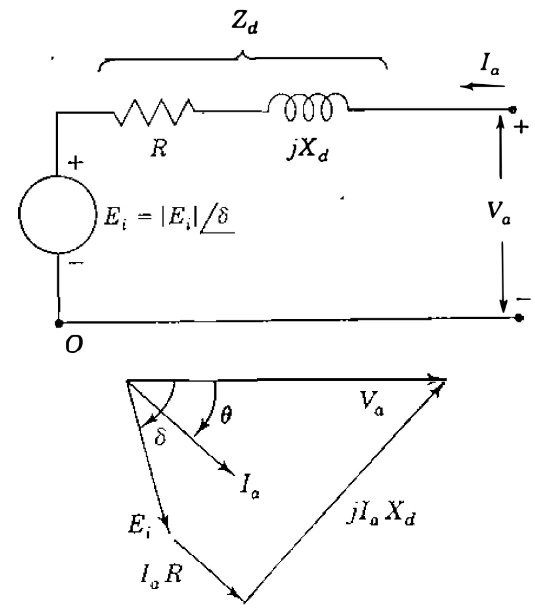


$$\bar{V}_a = \bar{E}_i - \bar{I}_a\bar{Z}_d = \bar{E}_i - \bar{I}_aR - j\bar{I}_aX_d$$

Figure 9(a)

Equivalent circuits for a synchronous generator and phasor diagrams of an over-excited generator delivering lagging current $\bar{I}_a$

Motor



$$\bar{V}_a = \bar{E}_i + \bar{I}_a\bar{Z}_d = \bar{E}_i + \bar{I}_aR + j\bar{I}_aX_d$$

Figure 9(b)

Equivalent circuits for a synchronous motor and phasor diagrams of an under-excited motor drawing lagging current $\bar{I}_a$

Synchronous reactance and equivalent circuits

Except for the case of an isolated generator supplying its own load, **most synchronous machines are connected to large interconnected power systems.**

In this case the **terminal voltage $\bar{V}_a$** (soon to be called $\bar{V}_t$ for emphasis) **is not altered by machine loading**.

The point of connection is therefore called an **infinite bus**, which means that **its voltage remains constant and no frequency change occurs** regardless of changes made in operating the synchronous machine.

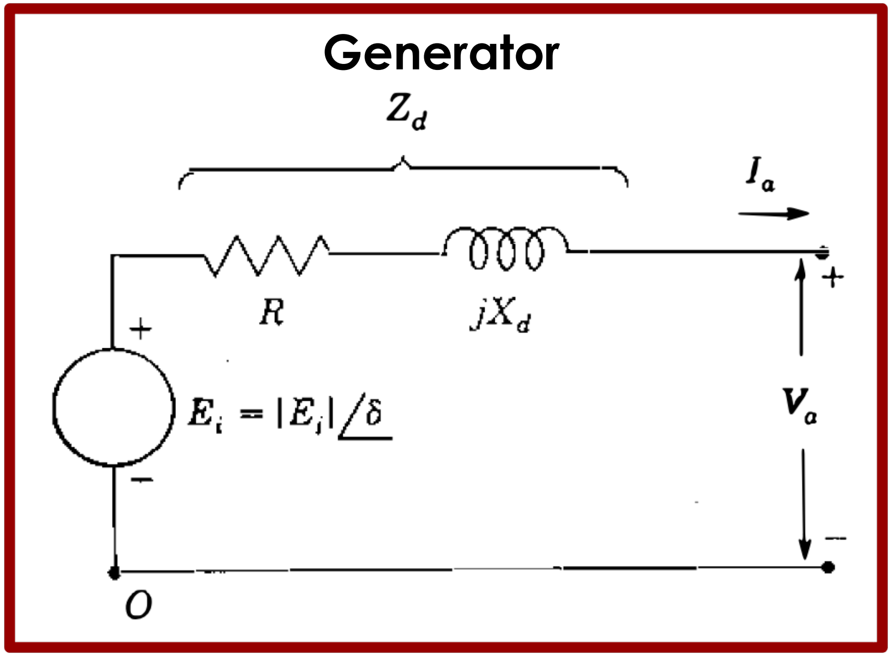


Figure 9(a)

Outline

Description of the synchronous machine

Three phase generation

Synchronous reactance and equivalent circuits

Real and reactive power control

The two-axis machine model

Transient and sub-transient effects

Real and reactive power control

When the synchronous machine is connected to an infinite bus, its speed and terminal voltage are fixed and unalterable.

Two controllable variables, however, are:
- The field current
- The mechanical torque on the shaft.

The variation of the **field current** I_f referred to as excitation system control, is applied to either a generator or a motor to supply or absorb a variable amount of **reactive power**.

Because **the synchronous machine runs at constant speed**, the only means of **varying the real power is through control of the torque imposed on the shaft by** either **the prime mover** in the case of a generator or the mechanical load in the case of a motor.

Important: it is not possible to change the active power of a machine by changing the excitation current.

Real and reactive power control

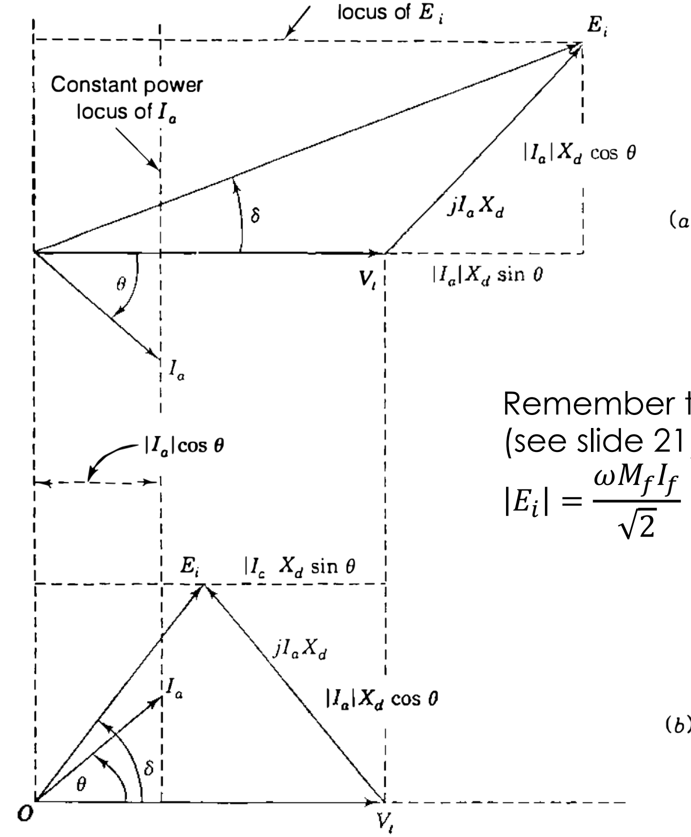
It is convenient to neglect resistance as we consider reactive power control of the round-rotor generator. Assume that the **generator is delivering power so that a certain angle δ** exists between the terminal voltage $\bar{V}_t$ and the generated voltage $\bar{E}_i$ of the machine.

The complex power delivered to the system by the generator is given in per unit by:

$$\bar{S} = P + jQ = \bar{V}_t \; \bar{I}_a^* \\ = |\bar{V}_t||\bar{I}_a|(\cos\theta + j\sin\theta) \qquad (4.1)$$

Equating real and imaginary quantities in this equation, we obtain:

$$P = |\bar{V}_t||\bar{I}_a|\cos\theta \qquad (4.2)$$
$$Q = |\bar{V}_t||\bar{I}_a|\sin\theta \qquad (4.3)$$



Remember that (see slide 21)
$$|E_i| = \frac{\omega M_f I_f}{\sqrt{2}}$$

Figure 10
Phasor diagrams showing:
(a) Over-excited generator delivering reactive power to the system;
(b) Under-excited generator receiving reactive power from the system.
The power delivered is the same in both cases (because $I_a\cos\theta$ is the same).

Real and reactive power control

Reactive power

Observations:

- Q is positive for lagging power factors since the angle θ is numerically positive.
- To maintain a certain power delivery P from the generator to the constant voltage system, $|\bar{I}_a|\cos\theta$ must remain constant.
- As we vary the DC field current I_f under these conditions, the generated voltage $\bar{E}_i$, varies proportionally but always so as to keep $|\bar{I}_a|\cos\theta$ constant

Different excitation conditions:

- Normal excitation is defined as the condition when $|\bar{E}_i|\cos\delta = |\bar{V}_t|$
- Over-excited when $|\bar{E}_i|\cos\delta > |\bar{V}_t|$
- Under-excited when $|\bar{E}_i|\cos\delta < |\bar{V}_t|$.

- For the condition of **over-excited generator**, this supplies reactive power Q to the system. Thus, from the system viewpoint the machine is acting like a **capacitor**.
- An **under-excited generator** supplying the same amount of real power and a leading current to the system draws reactive power from the system and in this respect acts like an **inductor**.

Real and reactive power control

Reactive power

Figure 11 shows **overexcited and under-excited synchronous motors** drawing the same real power at the same terminal voltage.

- The **over-excited motor** draws leading current and acts like a capacitive circuit when viewed from the network to which it supplies reactive power.
- The **under-excited motor** draws lagging current, absorbs reactive power, and is acting like an inductive circuit when viewed from the network.

In general:

- **Over-excited generators and motors supply reactive power to the system**
- **Under-excited generators and motors absorb reactive power from the system.**

Motor

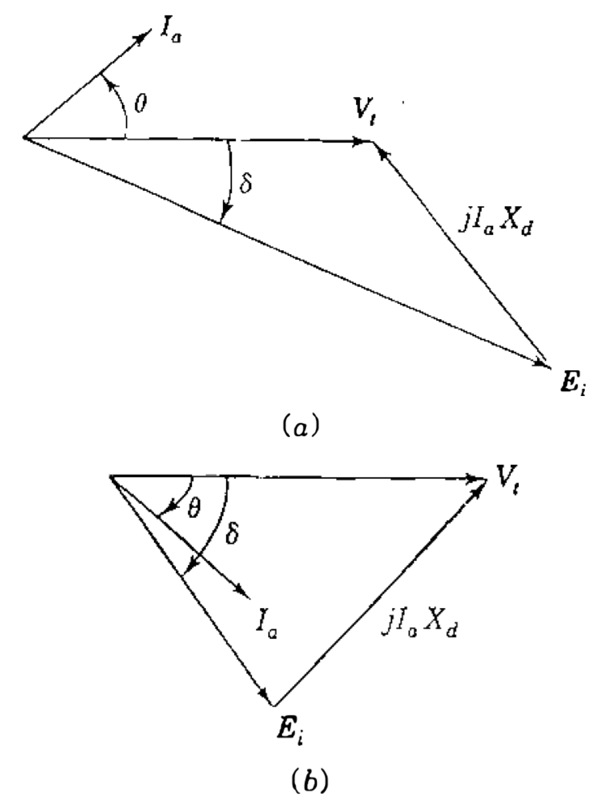


Figure 11

Phasor diagrams showing:
(a) Over-excited motor
(b) Under-exited motor
Both drawing current I_a and constant power at constant terminal voltage.

Real and reactive power control

Active power

The real power P is controlled by opening or closing the valves through which steam (or water) enters a turbine in order to change the **torque**.

The machine goes through the following steps:

1. If the power input to the generator is increased, **the rotor speed will start to increase**
2. If the field current I_f and hence $|\bar{E}_i|$ are held constant, **the angle $\boldsymbol{\delta}$ between $\bar{\boldsymbol{E}}_{\boldsymbol{i}}$ and $\bar{\boldsymbol{V}}_{\boldsymbol{t}}$ will increase**.
3. Increasing δ **results in a larger $|\bar{\boldsymbol{I}}_{\boldsymbol{a}}| \cos \boldsymbol{\theta}$**, as may be seen by rotating the phasor $\bar{E}_i$ counter-clockwise in Figs. 10(a) and 10(b).
4. The generator with a larger δ delivers more power to the network and **develops an higher counter-torque** on the prime mover
5. The input from the prime mover is re-established at the speed corresponding to the frequency of the infinite bus

Real and reactive power control

Active power

The dependence of ***P* on the power angle $\boldsymbol{\delta}$ between $\boldsymbol{\bar{E}_i}$ and $\boldsymbol{\bar{V}_t}$** is also shown as follows. By considering :

$\bar{V}_t = |\bar{V}_t|\angle 0°$ and

$\bar{E}_i = |\bar{E}_i|\angle\delta$

where $\bar{V}_t$ and $\bar{E}_i$ are expressed in volts to neutral or in per unit, then:

$$\bar{I}_a = \frac{|\bar{E}_i|\angle\delta - |\bar{V}_t|}{jX_d} \quad \text{and} \quad \bar{I}_a^* = \frac{|\bar{E}_i|\angle -\delta - |\bar{V}_t|}{-jX_d} \tag{4.4}$$

Therefore, the complex power delivered to the system at the terminals of the generator is given by

$$\bar{S} = P + jQ = \bar{V}_t\bar{I}_a^* = \frac{(|\bar{V}_t||\bar{E}_i|\angle -\delta) - |\bar{V}_t|^2}{-jX_d} \tag{4.5}$$

$$\bar{S} = \frac{|\bar{V}_t||\bar{E}_i|(\cos\delta - j\sin\delta) - |\bar{V}_t|^2}{-jX_d} \tag{4.6}$$

Real and reactive power control

The complex power delivered to the system at the terminals of the generator is given by

$$\bar{S} = P + jQ = \frac{|\bar{V}_t||\bar{E}_i|(\cos\delta - j\sin\delta\,) - |\bar{V}_t|^2}{-jX_d}$$

The real and imaginary parts:

$$\boxed{P = \frac{|\bar{V}_t||\bar{E}_i|}{X_d}\sin\delta} \qquad \boxed{Q = \frac{|\bar{V}_t|}{X_d}(|\bar{E}_i|\cos\delta - |\bar{V}_t|\,)} \qquad (4.7)$$

These two equations (identical to those we derived for a lossless and shunt-less transmission line) shows clearly the following:

- The dependence of P on the power angle δ if $|\bar{E}_i|$ and $|\bar{V}_t|$ are constant.
- If P and $\bar{V}_t$ are constant, δ must decrease if $|\bar{E}_i|$ is increased by boosting the DC field excitation.
- With P constant, both an increase in $|\bar{E}_i|$ and a decrease in δ mean that Q will increase if it is already positive, or it will decrease in magnitude and perhaps become positive if Q is already negative before the field excitation is boosted.

Real and reactive power control

Summary

When the synchronous machine is connected to an infinite bus, its speed and terminal voltage are fixed and unalterable

Different excitation conditions:

- Normal excitation if $|\bar{E}_i|\cos\delta = |\bar{V}_t|$
- Over-excited when $|\bar{E}_i|\cos\delta > |\bar{V}_t|$
- Under-excited when $|\bar{E}_i|\cos\delta < |\bar{V}_t|$

Active and reactive power:

$$\boxed{P = \frac{|\bar{V}_t||\bar{E}_i|}{X_d}\sin\delta}$$

$$\boxed{Q = \frac{|\bar{V}_t|}{X_d}(|\bar{E}_i|\cos\delta - |\bar{V}_t|\,)}$$

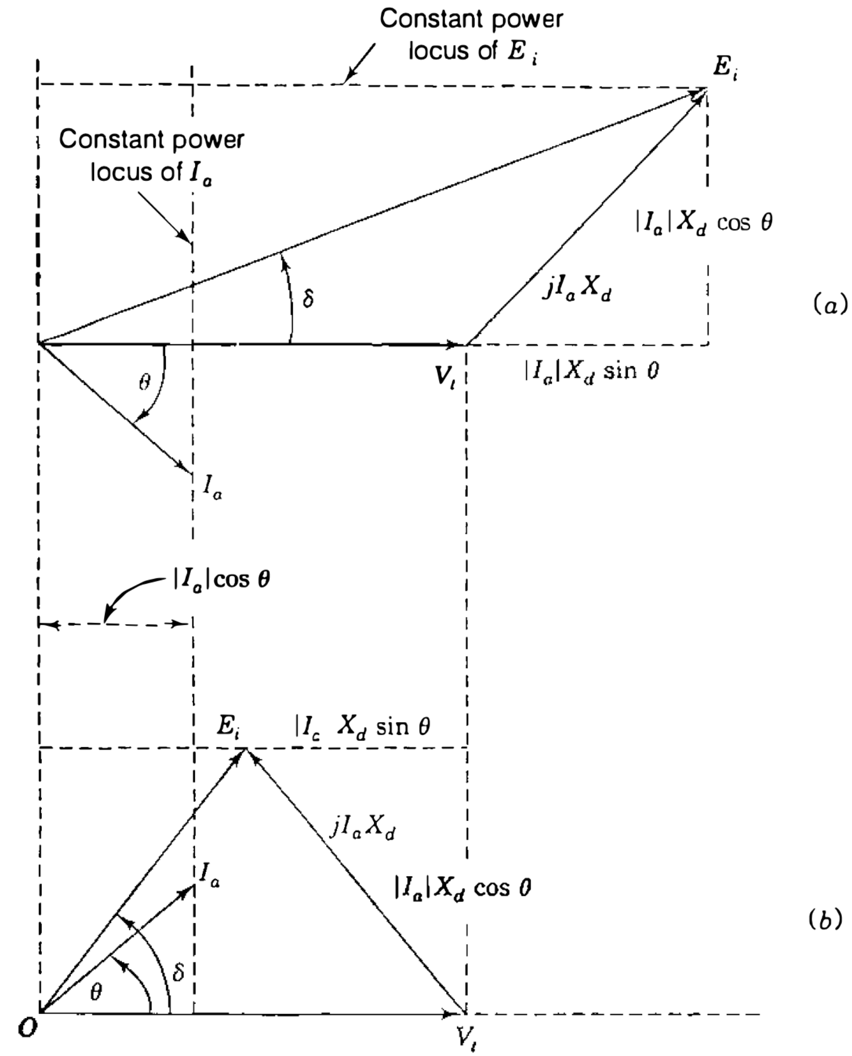


Figure 10

Outline

Description of the synchronous machine

Three phase generation

Synchronous reactance and equivalent circuits

Real and reactive power control

The two-axis machine model

Transient and sub-transient effects

The two axis machine model

The **round-rotor theory** already developed good results, but with the **hypothesis of for the steady-state**.

However, for **transient analysis** we need to consider a **two-axis model**.

The **two-axis model** will be introduced by means of the equations of **the salient-pole machine:**

- It these machines in which the air gap is much narrower along the direct axis than along the quadrature axis between poles.
  - As a direct consequence, during each revolution of the rotor, the self inductances L_{aa}, L_{bb}, L_{cc} of the stator windings, and the mutual inductances L_{ab}, L_{bc}, L_{ca}, are not constant in but **also vary as a function of the rotor angular displacement $\boldsymbol{\theta_d}$**
- Despite this difference, the other characteristic are similar to the round rotor machine:
  - Three symmetrically distributed armature windings ***a, b, c***
  - Field winding ***f*** on the rotor which produces a sinusoidal flux distribution around the air gap.
  - The field winding has constant self-inductance L_{ff}

The two axis machine model

The flux linkages of phases ***a, b***, and ***c*** are related to the currents by the inductances so that:

$$
\begin{aligned}
\lambda_a &= L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f \\
\lambda_b &= L_{ba}i_a + L_{bb}i_b + L_{cc}i_c + L_{bf}i_f \\
\lambda_c &= L_{ca}i_a + L_{cb}i_b + L_{cc}i_c + L_{cf}i_f
\end{aligned} \tag{5.1}
$$

These equations look the same as the one obtained before (2.1) but all the coefficient are variable, which makes them very hard to solve

Salient-pole machine

Self inductances

$(L_s > L_m > 0)$

$$
\begin{aligned}
L_{aa} &= L_s + L_m \cos 2\theta_d \\
L_{bb} &= L_s + L_m \cos 2(\theta_d - 2/3\pi) \\
L_{cc} &= L_s + L_m \cos 2(\theta_d - 4/3\pi)
\end{aligned}
$$

Mutual inductances

$(M_s > L_m > 0)$

$$
\begin{aligned}
L_{ab} &= L_{ba} = -M_s + L_m \cos 2(\theta_d - \pi/6) \\
L_{bc} &= L_{cb} = -M_s + L_m \cos 2(\theta_d - 3\pi/6) \\
L_{ca} &= L_{ac} = -M_s + L_m \cos 2(\theta_d - 5\pi/6)
\end{aligned}
$$

Armature/Field

$$
\begin{aligned}
L_{af} &= L_{fa} = M_f \cos 2\theta_d \\
L_{bf} &= L_{fb} = M_f \cos 2(\theta_d - 2/3\pi) \\
L_{cf} &= L_{fc} = M_f \cos 2(\theta_d - 4/3\pi)
\end{aligned}
$$

Round-rotor machine

Self inductances

$$
\begin{aligned}
L_{aa} &= L_s \\
L_{bb} &= L_s \\
L_{cc} &= L_s
\end{aligned}
$$

Mutual inductances

$$
\begin{aligned}
L_{ab} &= L_{ba} = -M_s \\
L_{bc} &= L_{cb} = -M_s \\
L_{ca} &= L_{ac} = -M_s
\end{aligned}
$$

Armature/Field

$$
\begin{aligned}
L_{af} &= L_{fa} = M_f \cos 2\theta_d \\
L_{bf} &= L_{fb} = M_f \cos 2(\theta_d - 2/3\pi) \\
L_{cf} &= L_{fc} = M_f \cos 2(\theta_d - 4/3\pi)
\end{aligned}
$$

The two axis machine model

- The equations of the salient-pole machine can be expressed in a simple form by **transforming the *a*, *b*, and *c* variables of the stator into corresponding sets of new variables, called the direct-axis**, **quadrature-axis**, and **zero-sequence** quantities which are distinguished by the subscripts ***d*, *q* and *0***: respectively.

- The idea of transforming these equations comes from the fact that **almost all parameters depend unequivocally on the displacement angle θ_d.**

- For this reason, ***d* and *q* are chosen as axes of reference system which rotates integrally with the rotor.**

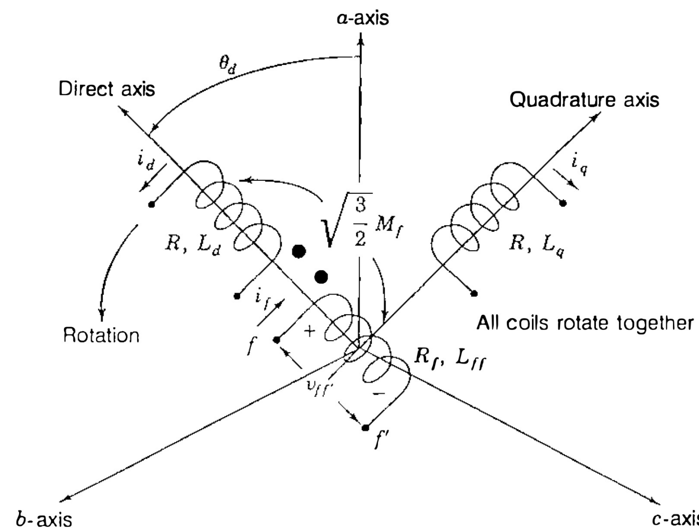


Figure 12

Representation of the salient-pole synchronous generator by armature-equivalent direct-axis and quadrature-axis coils rotating in synchronism with the field winding on the rotor.

The two axis machine model

The three stator currents i_a, i_b and i_c can be transformed into three equivalent currents, called the direct-axis current i_d, the quadrature-axis current i_q and the zero-sequence current i_0. The transformation is made by the matrix $\boldsymbol{P}$, called **Park's transformation**

$$\boldsymbol{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_d & \cos(\theta_d - 120°) & \cos(\theta_d - 240°) \\ \sin\theta_d & \sin(\theta_d - 120°) & \sin(\theta_d - 240°) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{5.2}$$

- The P-transformation defines a set of currents, voltages, and flux linkages for **three fictitious coils**, one of which is the stationary 0-coil.
- The other two coils are the ***d*-coil** and the ***q*-coil**, which rotate in synchronism with the rotor.

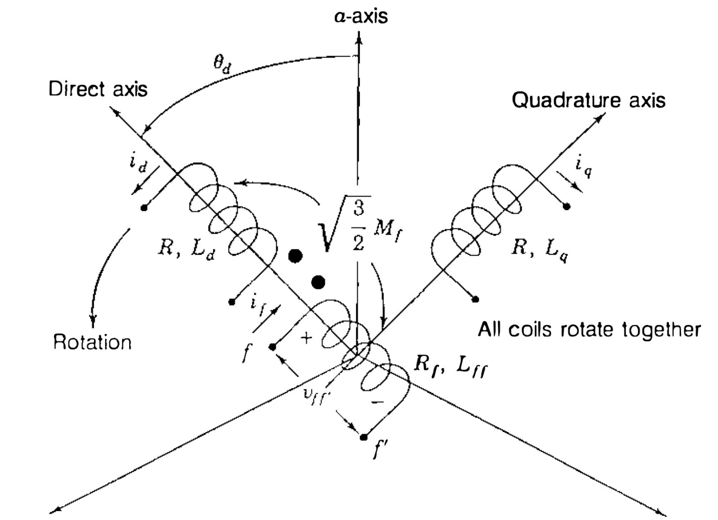


Figure 12

The d-coil and the q-coil have constant flux linkages with the field and any other windings which may exist on the rotor.

The two axis machine model

The currents, voltages, and flux linkages of phases $a, b,$ and c are transformed by P to $\boldsymbol{d}, \boldsymbol{q}$, and $\boldsymbol{0}$ variables as follows:

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} \tag{5.3}$$

The matrix $\boldsymbol{P}$ has the convenient property (called orthogonality) that its inverse $\boldsymbol{P}^{-1}$ equals its transpose $\boldsymbol{P}^{T}$, which is found simply by interchanging rows and columns in Eq. (5.2).

This property ensures that power in the $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ variables is not altered by $\boldsymbol{P}$.

By applying Park's transformation the resulting $d, q,$ and 0 flux-linkage and inductance equations are:

$$\lambda_d = L_d i_d + \sqrt{\frac{3}{2}}\, M_f i_f \tag{5.4}$$

$$\lambda_q = L_q i_q \tag{5.5}$$

$$\lambda_0 = L_0 i_0 \tag{5.6}$$

$$L_d = L_s + M_s + \frac{3}{2} L_m \tag{5.7}$$

$$L_q = L_s + M_s - \frac{3}{2} L_m \tag{5.8}$$

$$L_0 = L_s - 2M_s \tag{5.9}$$

The two axis machine model

How to obtain Eqs. (5.4)-(5.9):

To transform ***a-b-c*** stator flux linkages to ***d-q-0*** quantities by means of matrix ***P*** of Eq. (5.2), rearrange the flux-linkage expressions of Eq. (5.1):

$$\begin{aligned}\lambda_a &= L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f \\ \lambda_b &= L_{ba}i_a + L_{bb}i_b + L_{cc}i_c + L_{bf}i_f \\ \lambda_c &= L_{ca}i_a + L_{cb}i_b + L_{cc}i_c + L_{cf}i_f\end{aligned} \tag{5.1}$$

as follows:

$$\begin{bmatrix}\lambda_a \\ \lambda_b \\ \lambda_c\end{bmatrix} = \begin{bmatrix}L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc}\end{bmatrix}\begin{bmatrix}i_a \\ i_b \\ i_c\end{bmatrix} + \begin{bmatrix}L_{af} \\ L_{bf} \\ L_{cf}\end{bmatrix} i_f \tag{5.1b}$$

Now substitute for the ***a-b-c*** flux linkages and currents from Eqs. (5.3) to obtain

$$\mathbf{P}^{-1}\begin{bmatrix}\lambda_d \\ \lambda_q \\ \lambda_0\end{bmatrix} = \begin{bmatrix}L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc}\end{bmatrix}\mathbf{P}^{-1}\begin{bmatrix}i_d \\ i_q \\ i_0\end{bmatrix} + \begin{bmatrix}L_{af} \\ L_{bf} \\ L_{cf}\end{bmatrix} i_f \tag{5.1b}$$

The two axis machine model

Eq. (5.1b) can be multiplied by $\boldsymbol{P}$ as follow to obtain:

$$\mathbf{P}\,\mathbf{P}^{-1}\begin{bmatrix}\lambda_d\\ \lambda_q\\ \lambda_0\end{bmatrix} = \mathbf{P}\begin{bmatrix}L_{aa} & L_{ab} & L_{ac}\\ L_{ba} & L_{bb} & L_{bc}\\ L_{ca} & L_{cb} & L_{cc}\end{bmatrix}\mathbf{P}^{-1}\begin{bmatrix}i_d\\ i_q\\ i_0\end{bmatrix} + \mathbf{P}\begin{bmatrix}L_{af}\\ L_{bf}\\ L_{cf}\end{bmatrix} i_f$$

$$\begin{bmatrix}\lambda_d\\ \lambda_q\\ \lambda_0\end{bmatrix} = \mathbf{P}\begin{bmatrix}L_{aa} & L_{ab} & L_{ac}\\ L_{ba} & L_{bb} & L_{bc}\\ L_{ca} & L_{cb} & L_{cc}\end{bmatrix}\mathbf{P}^{-1}\begin{bmatrix}i_d\\ i_q\\ i_0\end{bmatrix} + \mathbf{P}\begin{bmatrix}L_{af}\\ L_{bf}\\ L_{cf}\end{bmatrix} i_f \tag{5.1b}$$

Now, as already explained, the inductances matrix is variable with θ_d in the following way:

Salient-pole machine

Self inductances

$(L_s > L_m > 0)$

$$L_{aa} = L_s + L_m \cos 2\theta_d$$
$$L_{bb} = L_s + L_m \cos 2(\theta_d - 2/3\pi)$$
$$L_{cc} = L_s + L_m \cos 2(\theta_d - 4/3\pi)$$

Mutual inductances

$(M_s > L_m > 0)$

$$L_{ab} = L_{ba} = -M_s + L_m \cos 2(\theta_d - \pi/6)$$
$$L_{bc} = L_{cb} = -M_s + L_m \cos 2(\theta_d - 3\pi/6)$$
$$L_{ca} = L_{ac} = -M_s + L_m \cos 2(\theta_d - 5\pi/6)$$

Armature/Field

$$L_{af} = L_{fa} = M_f \cos 2\theta_d$$
$$L_{bf} = L_{fb} = M_f \cos 2(\theta_d - 2/3\pi)$$
$$L_{cf} = L_{fc} = M_f \cos 2(\theta_d - 4/3\pi)$$

The two axis machine model

The matrix form of:

Salient-pole machine

Self inductances

$(L_s > L_m > 0)$

$$L_{aa} = L_s + L_m \cos 2\theta_d$$
$$L_{bb} = L_s + L_m \cos 2(\theta_d - 2/3\pi)$$
$$L_{cc} = L_s + L_m \cos 2(\theta_d - 4/3\pi)$$

Mutual inductances

$(M_s > L_m > 0)$

$$L_{ab} = L_{ba} = -M_s + L_m \cos 2(\theta_d - \pi/6)$$
$$L_{bc} = L_{cb} = -M_s + L_m \cos 2(\theta_d - 3\pi/6)$$
$$L_{ca} = L_{ac} = -M_s + L_m \cos 2(\theta_d - 5\pi/6)$$

Armature/Field

$$L_{af} = L_{fa} = M_f \cos 2\theta_d$$
$$L_{bf} = L_{fb} = M_f \cos 2(\theta_d - 2/3\pi)$$
$$L_{cf} = L_{fc} = M_f \cos 2(\theta_d - 4/3\pi)$$

Is the following:

$$\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} = (L_s + M_s)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - M_s\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} +$$

$$-L_m\begin{bmatrix} -cos\ 2\theta_d & cos\ 2(\theta_d + \frac{\pi}{6}) & cos\ 2(\theta_d + \frac{5\pi}{6}) \\ cos\ 2(\theta_d + \frac{\pi}{6}) & -cos\ 2(\theta_d + \frac{2}{3\pi}) & cos\ 2(\theta_d + \frac{\pi}{2}) \\ cos\ 2(\theta_d + \frac{5\pi}{6}) & cos\ 2(\theta_d + \frac{\pi}{2}) & cos\ 2(\theta_d + \frac{2}{3\pi}) \end{bmatrix} \tag{5.10}$$

The two axis machine model

Finally by introducing Eq.(5.10) and the definition of $\boldsymbol{P}$, $\boldsymbol{P^T}$ into equation (5.1b):

$$\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} = (L_s + M_s)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - M_s\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - L_m\begin{bmatrix} -cos\,2\theta_d & cos\,2(\theta_d + \frac{\pi}{6}) & cos\,2(\theta_d + \frac{5\pi}{6}) \\ cos\,2(\theta_d + \frac{\pi}{6}) & -cos\,2(\theta_d + \frac{2}{3\pi}) & cos\,2(\theta_d + \frac{\pi}{2}) \\ cos\,2(\theta_d + \frac{5\pi}{6}) & cos\,2(\theta_d + \frac{\pi}{2}) & cos\,2(\theta_d + \frac{2}{3\pi}) \end{bmatrix} \quad (5.10)$$

$$\boldsymbol{P} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\theta_d & \cos(\theta_d - 120°) & \cos(\theta_d - 240°) \\ \sin\theta_d & \sin(\theta_d - 120°) & \sin(\theta_d - 240°) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \quad (5.2)$$

$$\boldsymbol{P}^{-1} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\theta_d & \sin\theta_d & 1/\sqrt{2} \\ \cos(\theta_d - 120°) & \sin(\theta_d - 120°) & 1/\sqrt{2} \\ \cos(\theta_d - 240°) & \sin(\theta_d - 240°) & 1/\sqrt{2} \end{bmatrix} \quad (5.2b)$$

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = \mathbf{P}\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}\mathbf{P}^{-1}\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \mathbf{P}\begin{bmatrix} L_{af} \\ L_{bf} \\ L_{cf} \end{bmatrix} i_f \qquad (5.1b)$$

We obtain Eqs. (5.4)-(5.9):

$$\lambda_d = L_d i_d + \sqrt{\frac{3}{2}}\, M_f i_f \quad (5.4) \qquad L_d = L_s + M_s + \frac{3}{2}L_m \quad (5.7)$$

$$\lambda_q = L_q i_q \quad (5.5) \qquad L_q = L_s + M_s - \frac{3}{2}L_m \quad (5.8)$$

$$\lambda_0 = L_0 i_0 \quad (5.6) \qquad L_0 = L_s - 2M_s \quad (5.9)$$

The two axis machine model

The flux linkages of the field are still given by equation obtained for the round-rotor machine. $\lambda_f = L_{ff}I_f + \sqrt{3/2}\,M_f i_d$

If we report **all the flux equations**:

$$\lambda_d = L_d i_d + \sqrt{\frac{3}{2}}\,M_f i_f \quad (5.10)$$

$$\lambda_q = L_q i_q \quad (5.11)$$

$$\lambda_0 = L_0 i_0 \quad (5.12)$$

$$\lambda_f = L_{ff}I_f + \sqrt{3/2}\,M_f i_d \quad (5.13)$$

Where:

- $\boldsymbol{L_d}$ is called the direct-axis inductance and defined by Eq.(5.7)
- $\boldsymbol{L_q}$ is called the quadrature-axis inductance and defined by Eq.(5.8)
- $\boldsymbol{L_0}$ is known as the zero-sequence inductance and defined by Eq.(5.9)
- L_s and M_s have the same meanings as before
- L_m is a positive number

We can note that they have **constant inductance coefficients**, and thus are quite simple to use.

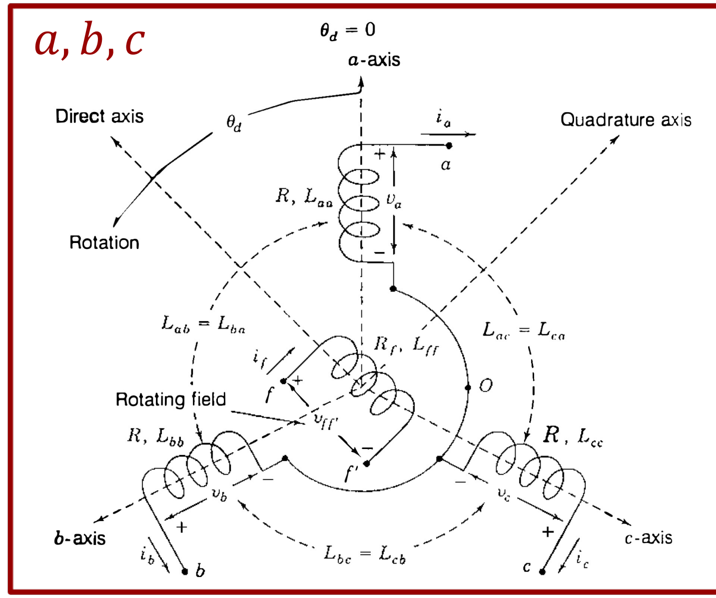
The two axis machine model

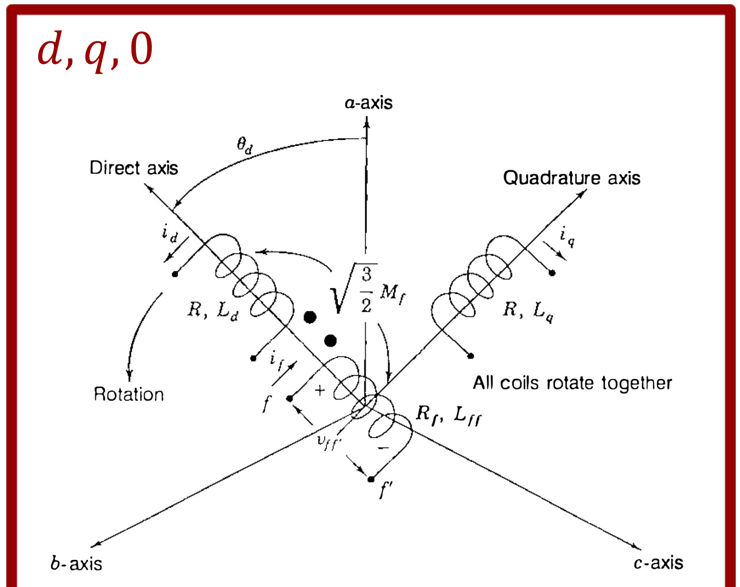
Obs#1:
i_d and i_q are stationary with respect to the rotor.

Obs#2:
$\boldsymbol{d}$ and $\boldsymbol{f}$ can be considered to act like two coupled coils, stationary with respect to each other as they rotate sharing the mutual inductance $3/2M_f$.

Obs#3:
$\boldsymbol{d}$ does not couple magnetically with $\boldsymbol{q}$ which lags the $\boldsymbol{d}$-axis in space by 90°

Obs#4:
The zero-sequence inductance L_0 is associated with a stationary fictitious armature coil with no coupling to any other coils. Under balanced conditions this coil carries no current, and therefore we omit it from further discussion.





The two axis machine model

Using the voltage polarities and current directions of Fig.(***a, b, c***), let us write the terminal-voltage equations for the armature windings of the salient-pole machine:

$$v_a = -Ri_a - \frac{d\lambda_a}{dt} \tag{5.14}$$

$$v_b = -Ri_b - \frac{d\lambda_b}{dt} \tag{5.15}$$

$$v_c = -Ri_c - \frac{d\lambda_c}{dt} \tag{5.16}$$

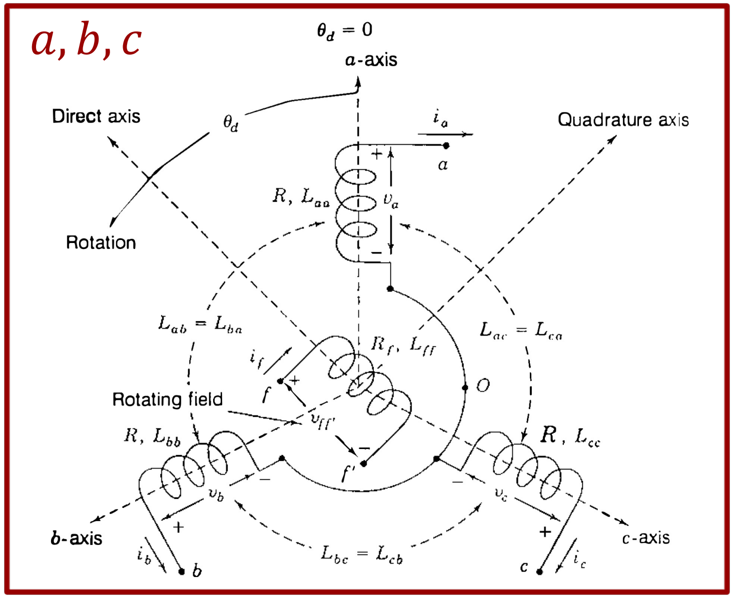
A much simpler set of equations for the voltages v_d, v_q, v_0 shown in Fig.(***d, q, 0***) is found by employing the P-transformation.

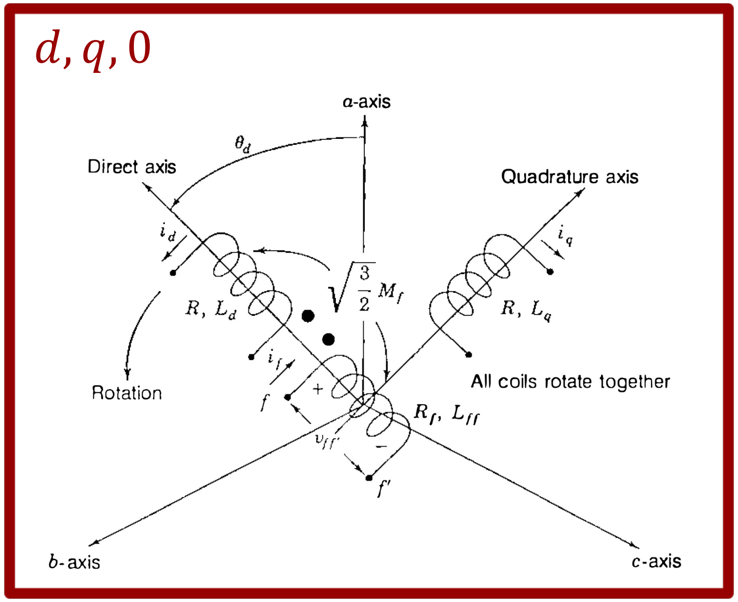
$$v_d = -Ri_d - \frac{d\lambda_d}{dt} - \omega\lambda_q \tag{5.17}$$

$$v_q = -Ri_q - \frac{d\lambda_q}{dt} + \omega\lambda_d \tag{5.18}$$

$$v_0 = -Ri_0 - \frac{d\lambda_0}{dt} \tag{5.19}$$

where ω is the rotational speed $d\theta_d/dt$.





The two axis machine model

Equation (2.18) for the field winding

$$v_{ff'} = R_f i_f + \frac{d\lambda_f}{dt}$$

is not subject to P-transformation, and so arranging the ***d, q, 0*** **flux-linkage and voltage equations** according to their axes gives the following equations:

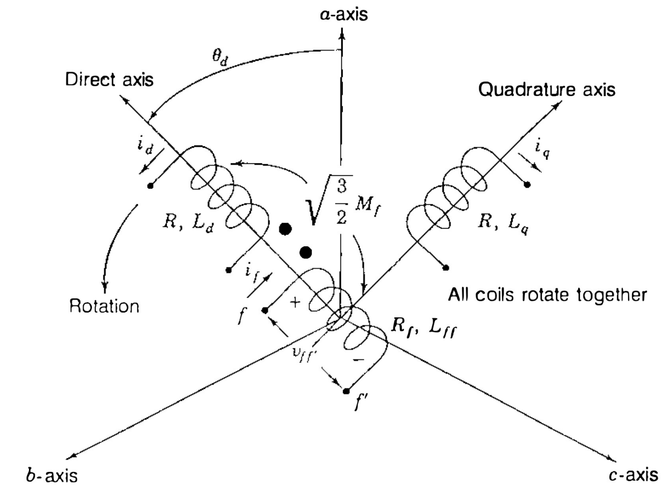
d-axis

$$\left.\begin{aligned}
\lambda_d &= L_d i_d + \sqrt{3/2}\, M_f i_f \\
\lambda_f &= \sqrt{3/2}\, M_f i_d + L_{ff}\, i_f \\
v_d &= -R i_d - \frac{d\lambda_d}{dt} - \omega\lambda_q \\
v_{ff'} &= R_f i_f + \frac{d\lambda_f}{dt}
\end{aligned}\right\} \quad (5.20)$$

q-axis

$$\left.\begin{aligned}
\lambda_q &= L_q i_q \\
v_q &= -R i_q - \frac{d\lambda_q}{dt} + \omega\lambda_d
\end{aligned}\right\} \quad (5.21)$$

$d, q, 0$



0-axis

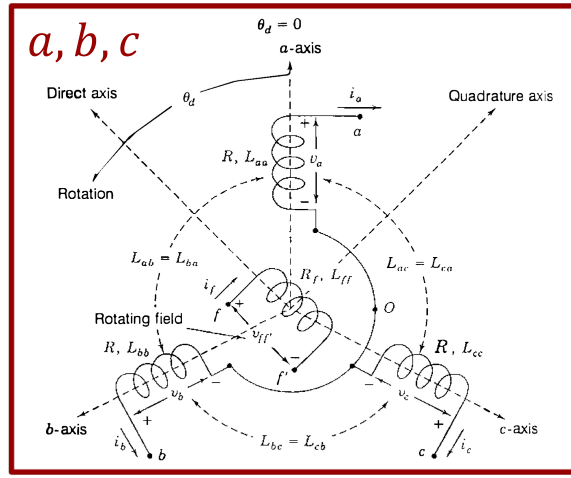
$$v_0 = -R i_0 - \frac{d\lambda_0}{dt} \qquad (5.22)$$

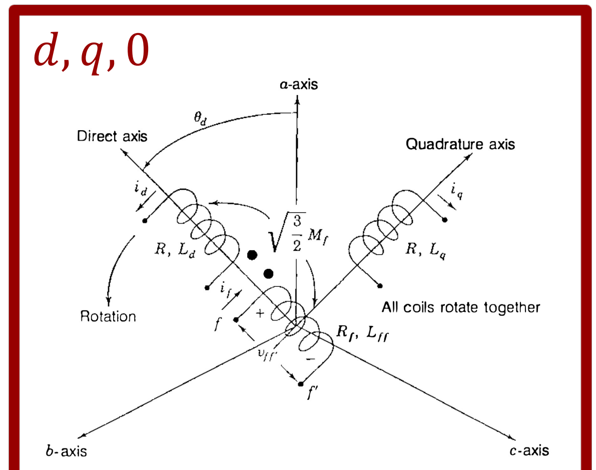
Equations involving i_o and λ_0 are not of interest under balanced conditions

The two axis machine model

Observations:

- The ***f*-coil is mutually coupled to the *d*-coil** on the ***d***-axis
- The fictitious ***q*-coil is not magnetically coupled** from the other two windings since the ***d***-axis and the ***q***-axis are spatially in quadrature.
- There is interaction between the two axes by means of the voltage sources $-\omega\lambda_q$ and $\omega\lambda_d$ which are rotational emfs or speed voltages internal to the machine due to the rotation of the rotor.
- **The speed voltage in the *d*-axis depends on λ_q**
- **The speed voltage in the *q*-axis depends on λ_d**
- No energy conversion could occur at standstill $(\omega = 0)$ since the field and the other ***d***-axis circuit would then act like a stationary transformer and the ***q***-axis circuit like an ordinary inductance coil.





The two axis machine model

To summarize, Park's transformation replaces the physical stationary windings of the armature by:

- **A direct-axis circuit** which rotates with the field circuit and is mutually coupled to it,
- **A quadrature-axis circuit** which is displaced 90° from the ***d***-axis, and thus has no mutual inductance with the field or other ***d***-axis circuits although it rotates in synchronism with them, and
- **A stationary stand-alone 0-coil** with no coupling to any other circuit, and thus is not shown in Fig. 13.

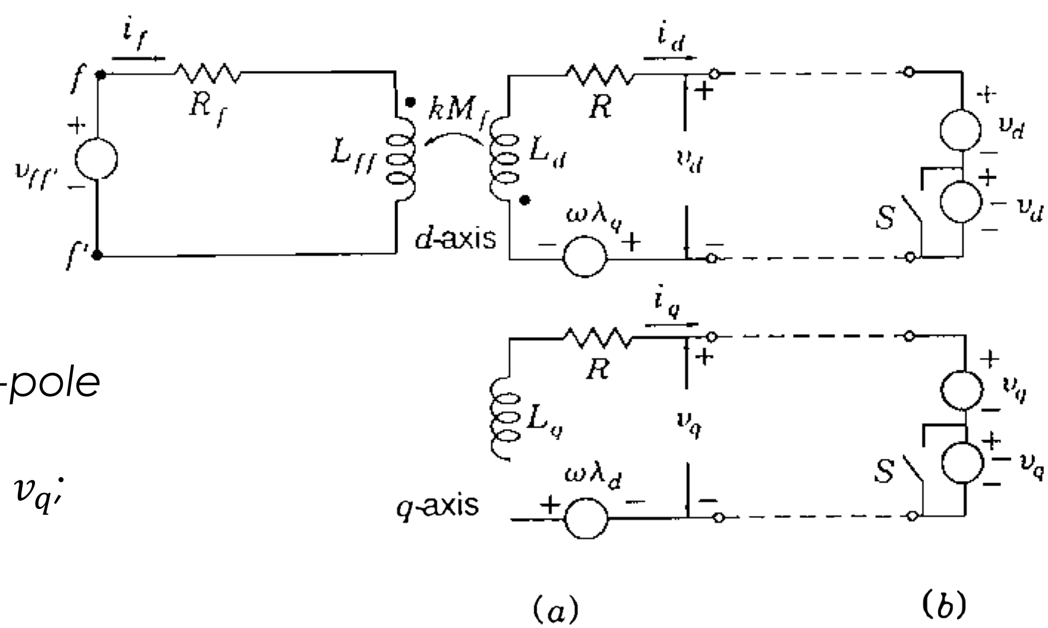


Figure 13

Equivalent circuit for the salient-pole synchronous generator:
(a) with terminal voltages v_d and v_q;
(b) with armature short-circuited.

Outline

Description of the synchronous machine

Three phase generation

Synchronous reactance and equivalent circuits

Real and reactive power control

The two-axis machine model

Transient and sub-transient effects

Transient and sub-transient effects

- **When a fault occurs** in a power network, **the current flowing is determined** by:
  - The internal emfs of the machines in the network,
  - The impedances of the machines in the network
  - The impedances in the network between machines and fault.

The current flowing in a synchronous machine immediately after the occurrence of a fault differs from the steady-state value of the fault current.

- This is because of the effect of the fault current in the armature on the flux generating the voltage in the machine.
- The current changes relatively slowly from its initial value to its steady-state value owing to **the changes in reactance of the synchronous machine.**
- Our immediate interest is in the inductance effective in the armature of the synchronous machine when a **three-phase short circuit** suddenly occurs at its terminals.

Transient and sub-transient effects

Before the fault occurs, suppose that the armature voltages are v_a, v_b, v_c and that these give rise to the voltages v_d, v_q, v_0 (according to Park)

- Figure 13(a) shows the voltages v_d and v_q at the terminals of the ***d***-axis and ***q***-axis equivalent circuits. The short circuit of phases ***a, b, c*** imposes the condition:

$$v_a = v_b = v_c = 0$$

- To simulate short-circuit conditions, the terminals of the ***d***-axis and ***q***-axis circuits in Fig.13(a) must also be shorted , as shown in Fig. 13(b).

- the switches S should be interpreted in a symbolic sense:
  - when the switches are both open, the sources $-v_d$ and $-v_q$ are in the circuit
  - when the switches are closed, those two sources are removed from the circuit.

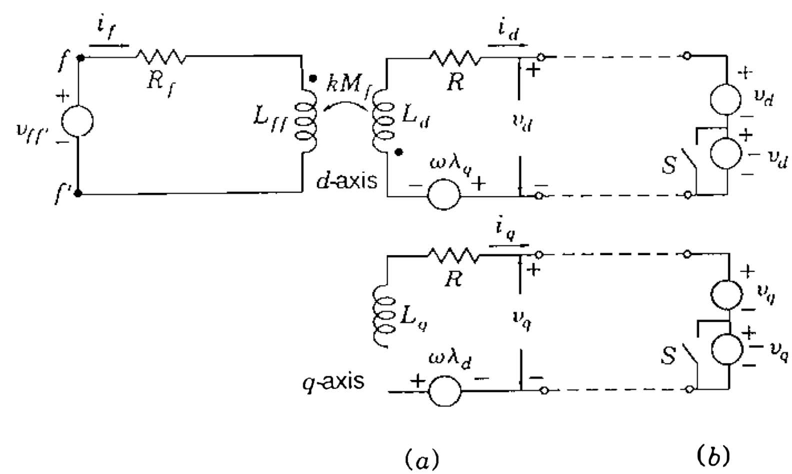


Figure 13

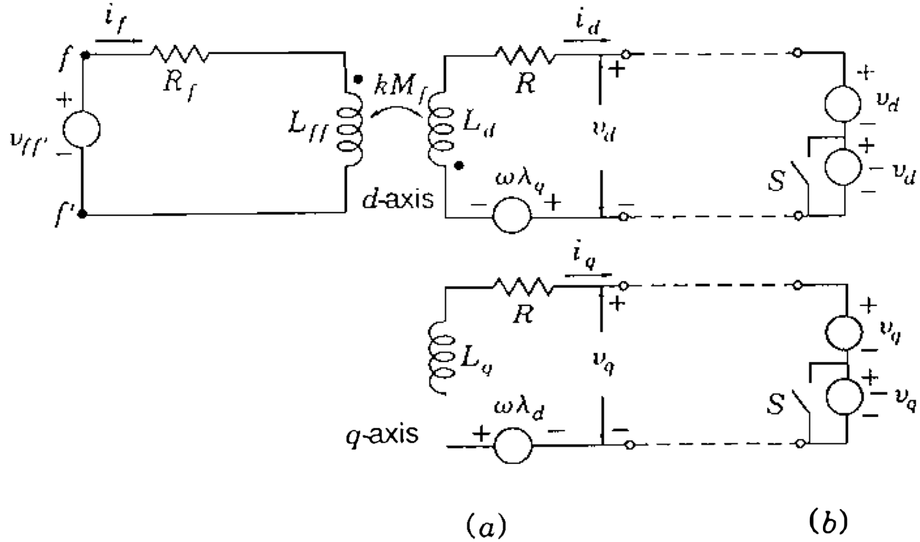
Transient and sub-transient effects

Hypothesis:

The rotor speed ω remains at its pre-fault steady-state value

Consequences:

- Eqs. (5.20) and (5.21) are linear and therefore the principle of superposition can be applied to the series-connected voltage sources.



With both **switches closed** in Fig. 13(b), we have the **steady-state operation** of the machine

Suddenly **opening the switches S** adds the voltage source $-v_d$ in series with the source v_d and $-v_q$ in series with the source v_q to **produce the required short circuits**.

Transient and sub-transient effects

By **superposition**, we can calculate the fault-induced changes of all variables by setting the external sources $v_{ff'}, v_d, v_q$ equal to zero and suddenly applying the voltages $-v_d$ and $-v_q$ to the unexcited rotating machine, as shown in Fig. 14.

- The internal speed voltages $-\omega\lambda_q$ and $\omega\lambda_d$ are initially zero because flux linkages with all coils are zero in Fig. 14 before applying the $-v_d$ and $-v_q$ sources.

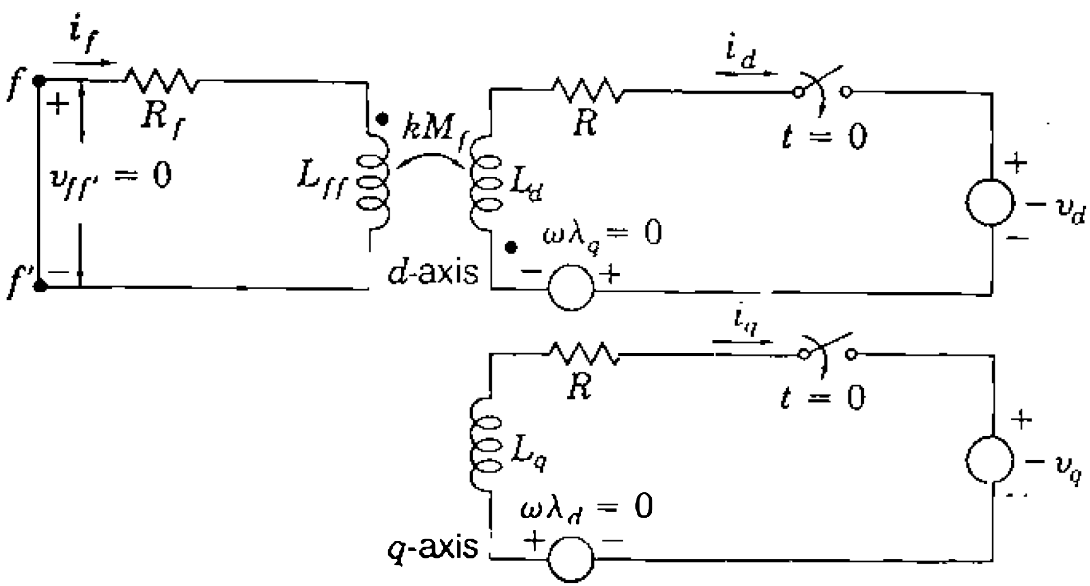


Figure 14
Equivalent circuit of salient-pole synchronous generator rotating at constant speed with field short-circuited. Closing switches at $t = 0$ corresponds to sudden application of short circuit to machine terminals

Transient and sub-transient effects

- The flux-linkage changes on the ***d***-axis of the machine are governed by Eq. (5.20), described in the previous chapter:

$$\left.\begin{aligned} \lambda_d &= L_d i_d + \sqrt{3/2}\, M_f i_f \\ \lambda_f &= \sqrt{3/2}\, M_f i_d \; + L_{ff}\, i_f \\ v_d &= -R i_d - \frac{d\lambda_d}{dt} - \omega \lambda_q \\ v_{ff'} &= R_f i_f + \frac{d\lambda_f}{dt} \end{aligned}\right\} \qquad (5.20)$$

which gives:

$$\Delta\lambda_d = L_d \Delta i_d + \sqrt{3/2}\, M_f \Delta i_f = L_d \Delta i_d + k M_f \Delta i_f \qquad (6.1)$$

$$\Delta\lambda_f = L_{ff} \Delta i_f + \sqrt{3/2}\, M_f \Delta i_d = L_{ff} \Delta i_f + k M_f \Delta i_d \qquad (6.2)$$

where Δ denotes incremental changes and $k = \sqrt{3/2}$.

Transient and sub-transient effects

Since the field winding is a closed physical winding, its flux linkages cannot change instantaneously according to the principle of constant flux linkages.

Therefore, setting $\Delta\lambda_f = 0$ in Eq. (6.2) gives $\Delta i_f = -(kM_f/L_{ff})\Delta i_d$ and substituting for Δi_f in the equation for $\Delta\lambda_d$ yields:

$$\Delta\lambda_d = \left[L_d - \frac{(kM_f)^2}{L_{ff}}\right]\Delta i_d \qquad (6.3)$$

The flux linkage per unit current In Eq. (6.3) defines the ***d*-axis transient inductance L_d'**, where

$$L_d' = \frac{\Delta\lambda_d}{\Delta i_d} = L_d - \frac{(kM_f)^2}{L_{ff}} \qquad (6.4)$$

Since $\frac{(kM_f)^2}{L_{ff}} > 0$ the direct-axis transient reactance $X_d' = \omega L_d'$ is always less than the direct-axis synchronous reactance $X_d = \omega L_d$.

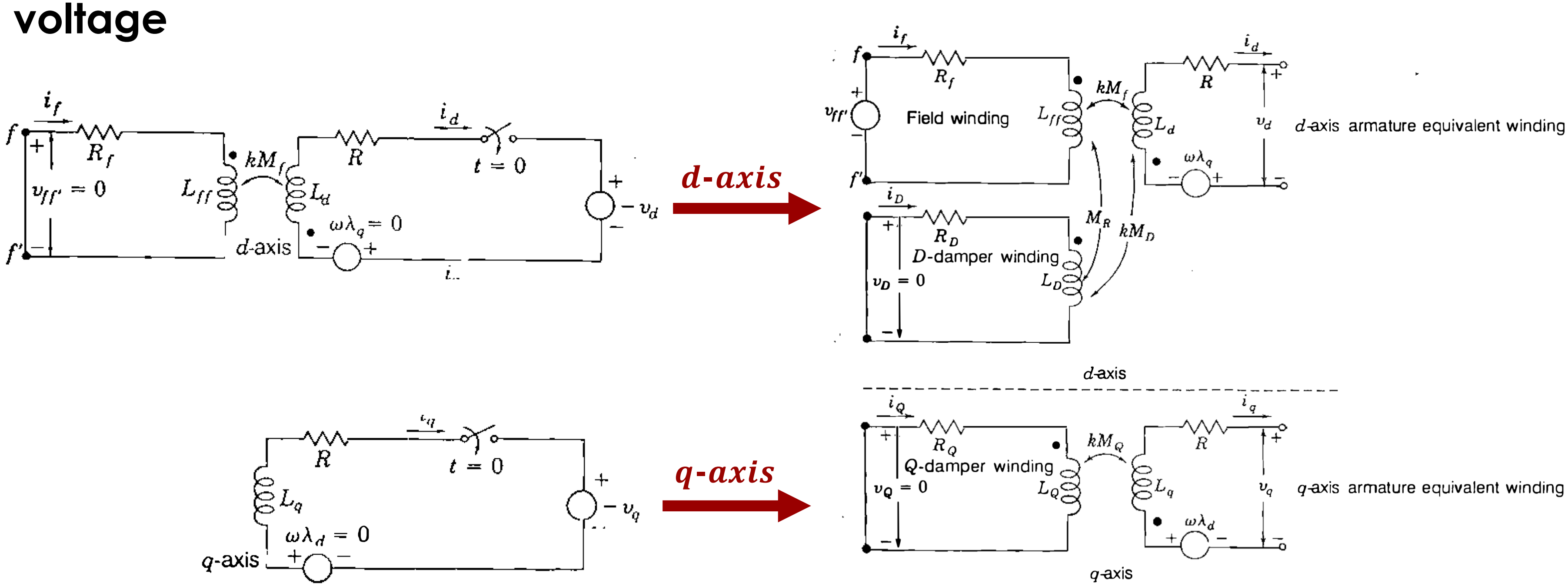
$$\boxed{X_d' < X_d}$$

Transient and sub-transient effects

In defining X_d', we assume that the field is the only physical rotor winding.

- Most **salient-pole machines** of practical importance have damper windings consisting of shorted copper bars through the pole faces of the rotor
- In a **round-rotor machine**, under short-circuit conditions eddy currents are induced in the solid rotor as if in damper windings.

The effects of the eddy-current **damping circuits** can be represented by direct-axis and quadrature-axis **closed coils**, which are treated in very much the same way as the field winding except that they have **no applied voltage**



Transient and sub-transient effects

- Two new circuits:
  - closed ***d***-circuit, self-inductances L_D
  - closed ***q***-circuit, self-inductances L_Q

- **Steady state condition:**
  - The flux linkages are constant between all circuits on the same rotor axis.
  - ***D***- and **Q**-circuits are then passive and do not enter into steady-state analysis.

- **Short-circuit conditions:**
  - The initial ***d***-axis flux-linkage changes resulting from sudden shorting of the synchronous machine with damper-winding effects.

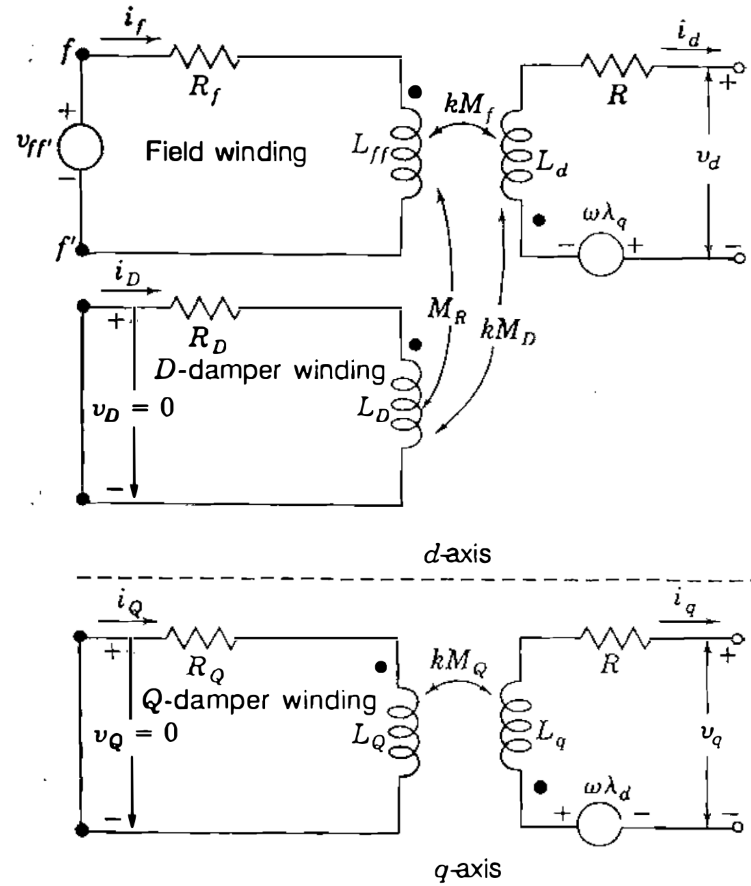


Figure 15

Equivalent circuit of the salient-pole synchronous generator with one field winding and two damper windings on the rotor.

Transient and sub-transient effects

The field and $\boldsymbol{D}$-damper circuits representing closed physical windings are mutually coupled to each other and to the $\boldsymbol{d}$-coil representing the armature along the direct axis.

There cannot be sudden change in the flux linkages of the closed windings, and so we can write for **the flux-linkage changes along the $\boldsymbol{d}$-axis**:

$$\Delta\lambda_d = L_d\Delta i_d + kM_f\Delta i_f + kM_D\Delta i_D \tag{6.5}$$

$$\Delta\lambda_f = kM_f\Delta i_d + L_{ff}\Delta i_f + M_r\Delta i_D = 0 \tag{6.6}$$

$$\Delta\lambda_D = kM_D\Delta i_d + M_r\Delta i_f + L_D\Delta i_D = 0 \tag{6.7}$$

These **equations are similar to Equations (6.1) and (6.2) but they have extra terms** because of the additional self- and mutual inductances **associated with the $\boldsymbol{D}$-damper circuit**.

M_r relates to mutual coupling between rotor-based windings on the $\boldsymbol{d}$-axis and thus has no $k = \sqrt{3/2}\,M_f$ multiplier.

Transient and sub-transient effects

Solving Eqs. (6.6)and (6.7), for Δi_f and Δi_D in terms of Δi_d yields:

$$\Delta i_f = -\left[\frac{(kM_f)L_D-(kM_D)M_r}{L_{ff}L_D-M_r^2}\right]\Delta i_d \tag{6.8}$$

$$\Delta i_D = -\left[\frac{(kM_D)L_{ff}-(kM_f)M_r}{L_{ff}L_D-M_r^2}\right]\Delta i_d \tag{6.9}$$

and substituting these results into the $\Delta\lambda_d$ expression of Eq. (6.5) yields the direct-axis sub-transient inductance L_d'' defined by:

$$\frac{\Delta\lambda_d}{\Delta i_d} = L_d'' = L_d - k^2\left[\frac{M_f^2L_D+M_D^2L_{ff}-2M_fM_DM_r}{L_{ff}L_D-M_r^2}\right] \tag{6.10}$$

The direct-axis sub-transient reactance X_d'', defined as $X_d''= \omega L_d''$, is considerably smaller than X_d', which means that:

$$\boxed{X_d'' < X_d' < X_d}$$

Similar reactances can be defined for the $\boldsymbol{q}$-axis.

Transient and sub-transient effects

The synchronous machine has **different reactances** when it is subjected to short-circuit faults at its terminals.

Reactance X''_d

- Immediately upon occurrence of the short circuit
- Combines with an effective resistance determined by the damping circuits to define a direct-axis,
- Short-circuit sub-transient time-constant T''_d in the range of 0.03 s.
- Sub-transient period typically 3 to 4 cycles of system frequency

Reactance X'_d

- When the damper-winding currents decay to negligible levels
- Short-circuit transient time-constant T'_d
- Transient period and T'_d is of the order of 1 s

Reactance X_d

- Sustained steady-state conditions
- d- and q-axis reactances $X_d = \omega L_d$ and $X_q = \omega L_q$

The various reactances supplied by the machine manufacturers are usually expressed in per unit based on the nameplate rating of the machine while time constants or given in seconds.

Transient and sub-transient effects